



Blind image deconvolution via dispersion minimization

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Abstract

In linear image restoration, the point spread function of the degrading system is assumed known even though this information is usually not available in real applications. As a result, both blur identification and image restoration must be performed from the observed noisy blurred image. This paper presents a computationally simple iterative blind image deconvolution method which is based on non-linear adaptive filtering. The new method is applicable to minimum as well as mixed phase blurs. The noisy blurred image is assumed to be the output of a two-dimensional linear shift-invariant system with an unknown point spread function contaminated by an additive noise. The method passes the noisy blurred image through a two-dimensional finite impulse response adaptive filter whose parameters are updated by minimizing the dispersion. When convergence occurs, the adaptive filter provides an approximate inverse of the point spread function. Moreover, its output is an estimate of the unobserved true image. Experimental results are provided.

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1. Introduction

The purpose of image restoration is to reconstruct an unobservable “true” image from a degraded observation. Blur and observation noise are the main sources of degradation. An observed image can be written, ignoring additive noise, as the two-dimensional (2-D) convolution of the true image with a linear shift-invariant (LSI) blur, known as the point spread function (PSF). Restoration in the case of known blur, assuming the linear degradation model, is called *linear image restoration* and it has been investigated extensively in the last three decades giving rise to a variety of solutions including inverse filtering, Wiener filtering, least-squares (LS) filtering, Kalman filtering, and iterative deconvolution methods [1–4]. In many practical situations, however, the blur is unknown. Hence, both blur identification and image restoration must be performed from the degraded image. Restoration in the case of unknown blur is called *blind image restoration (deconvolution)*.

Kundur and Hatzinakos [5,6] provide excellent tutorials which categorize blind image deconvolution methods into two major groups: (i) those which estimate the PSF a priori independent of the true image so as to use it later with one of the linear image restoration methods, and (ii) those which estimate the PSF and the true image simultaneously. Algorithms belonging to the first class are computationally simple, but they are limited to situations in which the PSF has a special parametric form, and the true image has certain features. Algorithms belonging to the second class, which are computationally more complex, must be used for more general situations.

In this paper, a new iterative blind image deconvolution method that belongs to the second class is proposed. The method is based on non-linear adaptive filtering and is most applicable to six or less-bit gray scale images. The proposed method utilizes a cost function like all other iterative non-linear adaptive filtering methods in order to update the parameters of the adaptive filter. The constant-modulus (CM) cost [7,8], which is one of the most studied and implemented methods of blind adaptive equalization for data communications over dispersive channels, is used as the cost function. First, it is shown how the method can be extended to the 2-D case. Then, this 2-D extension is applied to the blind image deconvolution problem.

2. Problem statement

Consider the single-input single-output (SISO) discrete-time LSI system depicted in Fig. 1, in which $f(m, n)$, $h(m, n)$, $v(m, n)$, and $g(m, n)$ represent the (m, n) th pixel of the true image, the PSF of the degrading system, additive noise that is independent of $f(m, n)$, and the degraded image, respectively.

It is clear from Fig. 1 that the observed $M \times N$ noisy blurred image $g(m, n)$ can be written as

$$\begin{aligned} g(m, n) &= f(m, n) * h(m, n) + v(m, n) \\ &= \sum_{k=1}^M \sum_{l=1}^N f(k, l) h(m-k, n-l) + v(m, n) \end{aligned} \quad (1)$$

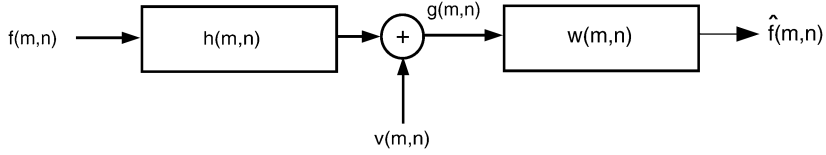


Fig. 1. General linear image deconvolution.

for $m = 1, \dots, M, n = 1, \dots, N$. In Eq. (1), $*$ denotes the 2-D linear convolution operator and $h(m, n)$, $(m, n) \in S_h$ assumes non-zero values only over S_h . Since blurs are usually modelled as 2-D finite impulse response (FIR) filters, S_h is a proper subset of the set of 2-D integers.

In linear image restoration, the PSF is given and the true image is estimated using one of the well-known linear image restoration algorithms. But, the PSF of the degrading system, $h(m, n)$, is usually unknown in most real applications. Hence, the true image must be estimated directly from the noisy blurred image using only partial information about the true image and the PSF. This process is called *blind image deconvolution*.

3. Image deconvolution via dispersion minimization

This section explains the proposed method in detail. In the remainder of the paper, the true image pixels are assumed to have (odd integer) gray levels $\pm 1, \pm 3, \dots, \pm(L - 1)$, where L is the number of gray levels in the true image, unless otherwise stated. Note that most of real images are 8-bit having 256 gray levels between 0 and 255. These images can be transformed to have gray levels $\pm 1, \pm 3, \dots, \pm(L - 1)$ by a uniform thresholding even though non-uniform thresholding based on the distribution of pixels in the true image may yield better results. General linear image deconvolution formulation and the CM cost on which the proposed method depends are explained next before the new method is described.

3.1. Supervised linear image deconvolution

Consider the general supervised image deconvolution problem depicted in Fig. 1, where the unobservable true image $f(m, n)$ is blurred by a PSF modelled as a 2-D FIR filter $h(m, n)$ with support $[-A, A] \times [-B, B]$, and is contaminated by an additive noise $v(m, n)$, which is independent of $f(m, n)$. The goal is to estimate the true image using a 2-D FIR filter $w(m, n)$ with support $[-C, C] \times [-D, D]$.

The notation $z(m, n) \sim z$ will be used to mean that the $z(m, n)$ are independent and identically distributed (i.i.d.) random variables for any m and n whose distribution is identical to that of some random variable z . For the sake of making the analysis simple, it will be assumed that (i) the true image is zero mean i.i.d. $f(m, n) \sim f$ with variance σ_f^2 , (ii) additive noise is zero mean i.i.d. $v(m, n) \sim v$ with variance σ_v^2 , (iii) the PSF is LSI with impulse response $h(m, n)$.

Let $\mathbf{f}(m, n)$, $\mathbf{v}(m, n)$ and $\mathbf{w}(m, n)$ denote the following lexicographically ordered vectors:

$$\mathbf{f}(m, n) := [f(m + P, n + Q), \dots, f(m - P, n - Q)]^T, \quad (2)$$

$$\mathbf{v}(m, n) := [v(m + C, n + D), \dots, v(m - C, n - D)]^T, \quad (3)$$

$$\mathbf{w} := [w(-C, -D), \dots, w(C, D)]^T, \quad (4)$$

where $P = A + C$, $Q = B + D$. Then, the estimate $\hat{f}(m, n)$ can be written as

$$\hat{f}(m, n) = \mathbf{f}^T(m, n)H\mathbf{w} + \mathbf{v}^T(m, n)\mathbf{w}, \quad (5)$$

where H is a suitable $(2P + 1)(2Q + 1) \times (2C + 1)(2D + 1)$ matrix whose entries are constructed from the PSF coefficients $h(m, n)$. In the absence of additive noise $v(m, n)$, Eq. (5) leads to support and zero conditions for perfect image deconvolution (PID), i.e., $\hat{f}(m, n) = f(m - m_0, n - n_0)$ for some fixed integer-valued vector (m_0, n_0) . PID requires the zero-forcing system impulse response

$$\mathbf{h}_{(m_0, n_0)} = H\mathbf{w} = [0, \dots, 0, 1, 0, \dots, 0]^T, \quad (6)$$

where the non-zero coefficient is in the (m_0, n_0) th position, where (m_0, n_0) must satisfy $-P \leq m_0 \leq P$ and $-Q \leq n_0 \leq Q$. In order to achieve this particular response, the system of linear equations described by $\mathbf{h}_{(m_0, n_0)} = H\mathbf{w}$ must have a solution. For PID under arbitrary (m_0, n_0) , H must be full row rank which implies that H must have at least as many columns as rows. Hence,

$$(2C + 1)(2D + 1) \geq (2P + 1)(2Q + 1). \quad (7)$$

From Eq. (7), it is clear that no 2-D FIR filter can perfectly cancel out the effect of a non-trivial blur even in the absence of additive noise since the row dimension of H always exceeds its column dimension (recall that $P = A + C$, $Q = B + D$).

In the presence of noise, it is common to minimize the expected value of the square of the recovery error $e(m, n)$ given by

$$e(m, n) := \hat{f}(m, n) - f(m - m_0, n - n_0) \quad (8)$$

for a particular choice of delay (m_0, n_0) . Using Eq. (5), $e(m, n)$ can be written as

$$\begin{aligned} e(m, n) &= \mathbf{f}^T(m, n)H\mathbf{w} + \mathbf{v}^T(m, n)\mathbf{w} - f(m - m_0, n - n_0) \\ &= \mathbf{f}^T(m, n)H\mathbf{w} + \mathbf{v}^T(m, n)\mathbf{w} - \mathbf{f}^T(m, n)\mathbf{h}_{(m_0, n_0)} \\ &= \mathbf{f}^T(m, n)(H\mathbf{w} - \mathbf{h}_{(m_0, n_0)}) + \mathbf{v}^T(m, n)\mathbf{w}. \end{aligned} \quad (9)$$

It was assumed that additive noise and the true image are i.i.d. and independent with respective variances σ_v^2 and σ_f^2 . Using this assumption yields

$$E[e^2(m, n)] = \|H\mathbf{w} - \mathbf{h}_{(m_0, n_0)}\|_2^2 \sigma_f^2 + \|\mathbf{w}\|_2^2 \sigma_v^2, \quad (10)$$

where $\|\cdot\|_2$ represents the l_2 -norm of a vector. Note that (10) is proportional to the true image-power normalized mean-squared error (MSE) J_{MSE} given by

$$J_{\text{MSE}} = \|H\mathbf{w} - \mathbf{h}_{(m_0, n_0)}\|_2^2 + \lambda \|\mathbf{w}\|_2^2, \quad (11)$$

where $\lambda = \sigma_v^2 / \sigma_f^2$. From Eq. (11), the adaptive filter parameter vector minimizing J_{MSE} is given by

$$\mathbf{w}^* = (H^T H + \lambda I)^{-1} H^T \mathbf{h}_{(m_0, n_0)}. \quad (12)$$

From (11) and (12) the optimal MSE is

$$J_{\text{MSE}}^* = \mathbf{h}_{(m_0, n_0)}^T (I - H(H^T H + \lambda I)^{-1} H^T) \mathbf{h}_{(m_0, n_0)}. \quad (13)$$

The MSE cost defined in (10) constitutes a well-known and useful measure of estimator performance. The performance of the new method can be quantified by comparing its MSE to the minimum achievable MSE given an identical true image and blur.

3.2. The CM cost

Traditional uses of the CM cost have all been one-dimensional (1-D). The CM cost term was introduced for blind equalization of communication signals over dispersive channels by Godard [7] and Treichler and Agee [8]. The reader is referred to Ref. [9] for a comprehensive introduction to the CM cost in the context of adaptive equalization. This section generalizes the CM cost for use in 2-D by reformulating the cost for a real-valued zero-mean true image $f(m, n)$ and a real-valued PSF $h(m, n)$ in additive zero-mean noise $v(m, n)$. It is assumed that each gray level of the true image is equally likely. The order of topics follows that in Ref. [9]. The CM cost is given by

$$\begin{aligned} J_{\text{CM}} &:= E[(\hat{f}^2(m, n) - \gamma)^2] = E[\hat{f}^4(m, n)] - 2\gamma E[\hat{f}^2(m, n)] + \gamma^2 \\ &= E[\hat{f}^4(m, n)] - 2\sigma_f^2 \kappa_f E[\hat{f}^2(m, n)] + \sigma_f^4 \kappa_f^2, \end{aligned} \quad (14)$$

where γ and κ_f are the dispersion constant and normalized kurtosis of the true image, respectively. They are defined by

$$\kappa_f := \frac{E[f^4(m, n)]}{(E[f^2(m, n)])^2}, \quad (15)$$

$$\gamma := \frac{E[f^4(m, n)]}{E[f^2(m, n)]}. \quad (16)$$

Note that $\gamma = \sigma_f^2 \kappa_f$. A detailed analysis of the CM cost for the 1-D case is given in Ref. [9]. Since the analysis for the 2-D case can be made similarly, it is omitted here to save space.

It is evident from Eq. (14) that the CM cost penalizes the deviations of $\hat{f}^2(m, n)$ from the dispersion constant γ . This interpretation of the CM cost explains why the proposed method is called *blind image deconvolution using dispersion minimization*. Table 1 gives the dispersion constant and normalized kurtosis of a zero mean uniformly distributed gray scale image for various gray levels.

Gradient descent (GD) methods are generally used to solve for CM estimators (dispersion minimizers) because closed form expressions do not usually exist. Since exact GD requires statistical knowledge of the degraded image, which is not available in real applications, stochastic GD methods are utilized. The algorithm that performs a stochastic GD minimization of J_{CM} is referred to as the *constant modulus algorithm or CMA* [9]:

$$\mathbf{w}_{j+1} = \mathbf{w}_j - \mu(\hat{f}_j^2(m, n) - \gamma)\hat{f}_j(m, n)\mathbf{g}(m, n). \quad (17)$$

Equation (17) is written in terms of the lexicographically ordered adaptive filter input vector at pixel (m, n) given by

$$\mathbf{g}(m, n) = [g(m + C, n + D), \dots, g(m - C, n - D)]^T, \quad (18)$$

Table 1
Dispersion constant and normalized kurtosis for a zero mean uniformly distributed image having different gray levels

Gray levels	γ	κ_f
2	1.00000	1.0000
4	8.20000	1.6400
8	37.00000	1.7169
16	152.2000	1.7905
32	613.0000	1.7977
64	2456.20	1.7994
128	9829.00	1.7999
256	39320.0	1.8000

the lexicographically ordered adaptive filter parameter vector \mathbf{w}_j at the j th iteration, the adaptive filter output $\hat{f}_j(m, n)$, a small positive step-size μ and the true image dispersion constant γ . Plotting the CM cost versus the adaptive filter parameters results in a surface called *the CM cost surface*. CMA attempts to minimize the CM cost by starting at some location on the surface and following the trajectory of the steepest descent.

3.3. Proposed algorithm

Consider the general blind image deconvolution problem depicted in Fig. 1. The model consists of a cascade connection of a linear degrading system $h(m, n)$ and a deconvolution filter $w(m, n)$. The nature of the PSF is determined by the type of the blur. The system tries to reconstruct the true image $f(m, n)$ given the observed blurred image $g(m, n)$. Equivalently, it is required to design a blind deconvolution filter $w(m, n)$ that is the (approximate) inverse of the unknown PSF, with the degrading system input being unobservable. The true image is assumed to have i.i.d. pixels which are uniformly distributed, and the PSF is assumed to have finite support. No other information is assumed.

The iterative blind deconvolution method is shown in Fig. 2, where the observed image $g(m, n)$ is applied to a 2-D FIR adaptive filter $w(m, n)$ which tries to remove the blur.

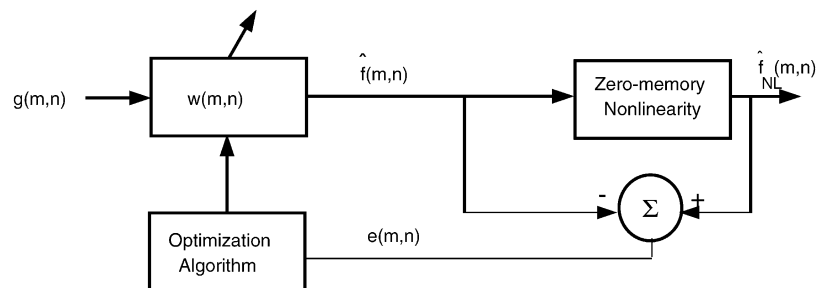


Fig. 2. Block diagram of the proposed method.

Thus, output of the adaptive filter at the j th iteration $\hat{f}_j(m, n)$ is an estimate of the true image given by

$$\hat{f}_j(m, n) = \sum_{k=-C}^C \sum_{l=-D}^D w_j(k, l)g(m-k, n-l) \quad (19)$$

for $m = 1, \dots, M, n = 1, \dots, N$, where $w_j(k, l)$ are the adaptive filter coefficients at the j th iteration for $-C \leq k \leq C, -D \leq l \leq D$. Ordinarily, the estimate $\hat{f}_j(m, n)$ at pixel (m, n) is not reliable enough. However, it may be used in an adaptive scheme to obtain a better estimate for the next pixel. If the true image $f(m, n)$ were known, then the difference between $\hat{f}_j(m, n)$ and $f(m, n)$ could be used to provide an efficient update of the filter parameters. In blind image deconvolution, however, the true image is unavailable. As in adaptive equalization, one possibility is to attempt to minimize the dispersion of $\hat{f}_j(m, n)$ using the CM cost J_{CM} . Since it is not possible to minimize an expected value directly, the method uses an instantaneous estimate of J_{CM} given by

$$J = \frac{1}{4}(\hat{f}^2(m, n) - \gamma)^2 \quad (20)$$

to obtain an implementable algorithm, where γ is the dispersion constant of the true image. Note that the function of the zero memory nonlinearity (the rightmost term in Fig. 2) is to produce an artificially generated desired image $\hat{f}_{\text{NL},j}(m, n)$ for the algorithm so that the difference between $\hat{f}_{\text{NL},j}(m, n)$ and the output of the adaptive filter $\hat{f}_j(m, n)$ can be used to update the adaptive filter coefficients. The zero memory nonlinearity is chosen such that this difference is equal to negative of the gradient of J .

The stochastic GD minimization is used to update the adaptive filter parameters. The derivative of J with respect to the adaptive filter parameters is needed in order to implement the stochastic GD minimization. Let $\mathbf{w}_j, \mathbf{g}(m, n)$ denote the following lexicographically ordered vectors:

$$\mathbf{w}_j := \begin{bmatrix} w_j(-C, -D) \\ w_j(-C, -D+1) \\ w_j(-C, -D+2) \\ \vdots \\ w_j(C, D) \end{bmatrix}, \quad \mathbf{g}(m, n) := \begin{bmatrix} g(m+C, n+D) \\ g(m+C, n+D-1) \\ g(m+C, n+D-2) \\ \vdots \\ g(m-C, n-D) \end{bmatrix} \quad (21)$$

for $m = 1, \dots, M, n = 1, \dots, N$. In Eq. (21), $w_j(m, n)$ stands for the (m, n) th coefficient of the adaptive filter at the j th iteration, where $-C \leq m \leq C$ and $-D \leq n \leq D$. Using vectors \mathbf{w}_j and $\mathbf{g}(m, n)$, the output of the adaptive filter for pixel (m, n) at the j th iteration can be written as

$$\hat{f}_j(m, n) = \mathbf{w}_j^T \mathbf{g}(m, n), \quad (22)$$

where T denotes vector transposition. Now, the derivative of J with respect to \mathbf{w}_j can be evaluated, which is given by

$$\frac{dJ}{d\mathbf{w}_j} = \frac{dJ}{d\hat{f}_j(m, n)} \frac{d\hat{f}_j(m, n)}{d\mathbf{w}_j} = [\hat{f}_j^2(m, n) - \gamma] \hat{f}_j(m, n) \mathbf{g}(m, n). \quad (23)$$

Hence, the adaptive filter is updated according to

$$\mathbf{w}_{j+1} = \mathbf{w}_j - \mu \frac{dJ}{d\mathbf{w}_j} = \mathbf{w}_j - \mu \phi(\hat{f}_j(m, n)) \mathbf{g}(m, n), \quad (24)$$

where μ is a small positive step-size (usually between 10^{-4} and 10^{-7} depending on the gray levels in the true image) that guarantees stability of the algorithm and $\phi(\hat{f}_j(m, n)) := [\hat{f}_j^2(m, n) - \gamma] \hat{f}_j(m, n)$ is called the *prediction error function*. The prediction error function $\phi(\cdot)$ has some interesting and important properties when the coefficients of the adaptive filter are near the global minimum of J_{CM} . The static and dynamic convergence analysis of the new method in the vicinity the global minimum of J_{CM} can be performed by using important features of $\phi(\cdot)$ which were presented in detail in Ref. [10]. Finally, the relationship between $\hat{f}_{\text{NL},j}$ and $f_j(m, n)$ will be provided before concluding this section. Note that $e_j(m, n)$ is related to J via

$$e_j(m, n) = -\frac{dJ}{d\hat{f}_j(m, n)} = [\gamma - \hat{f}_j^2(m, n)] \hat{f}_j(m, n),$$

where $e_j(m, n)$ is the error between $\hat{f}_{\text{NL},j}(m, n)$ and $\hat{f}_j(m, n)$ at the j th iteration. Hence, $\hat{f}_{\text{NL},j}(m, n)$ can be written in terms of $\hat{f}_j(m, n)$ as

$$\hat{f}_{\text{NL},j}(m, n) = \hat{f}_j(m, n) + e_j(m, n) = \hat{f}_j(m, n)[1 + \gamma - \hat{f}_j^2(m, n)].$$

Equations (22)–(24) constitute the proposed algorithm for blind deconvolution of noisy blurred images. Each iteration of the algorithm corresponds to processing a pixel in the blurred image. When convergence occurs, $\hat{f}_j(m, n)$ provides an estimate of the true image $f(m, n)$, and \mathbf{w}_j is an approximate inverse of the PSF.

4. Simulation results

1-D CMA analysis has shown that the performance of the CMA depends strongly on the source kurtosis, the signal-to-noise ratio (SNR), among other factors. Effectiveness of the new method is conjectured to depend on the normalized kurtosis (or the number of gray levels) of the true image and the blurred signal-to-noise ratio (BSNR). Results of two computer experiments are provided in this section in order to support this conjecture. The first experiment illustrates how the method performs in the absence of observation noise as true image kurtosis increases, while the second experiment demonstrates the behavior of the method as a function of BSNR for a fixed number of gray level in a true image.

The classical 8-bit gray-scale *Lena* image was chosen as a test image. Histogram equalization was performed on the test image which results in approximately uniformly distributed image. Then, its mean was subtracted from the histogram equalized image yielding a zero-mean uniformly distributed image. Finally, uniform quantizations having different step-sizes were applied to the zero-mean uniformly distributed image to obtain true images having different gray levels which fulfill most of the assumptions made about the true image.

In order to obtain artificially generated blurred images, a 5×5 scatter blur with parameter $\beta = 2$ whose coefficients are given by

$$h(m, n) = \frac{K}{(\beta^2 + (m^2 + n^2))^{3/2}} \quad (25)$$

was applied to the true images, where the constants K are chosen such that $\sum_m \sum_n h(m, n) = 1$ to preserve the mean value of the true image. Zero-mean Gaussian noises were added to the blurred images to get observed noisy blurred images. In all experiments a 5×5 support was used for the adaptive filter.

In image restoration studies, the degradation modelled by blurring and additive noise is referred to in terms of the metric BSNR. This metric for a zero mean $M \times N$ image is given by [4]

$$\text{BSNR} = 10 \log_{10} \left\{ \frac{\frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N z^2(m, n)}{\sigma_v^2} \right\}, \quad (26)$$

where $z(m, n)$ is the noise free blurred image and σ_v^2 is the additive noise variance. Performance of the method was tested at several BSNRs by adjusting the noise variance σ_v^2 .

For the purpose of objectively testing the performance of linear image restoration algorithms, the improvement in SNR (ISNR) is often used. This metric is given by [4]

$$\text{ISNR} = 10 \log_{10} \left\{ \frac{\sum_{m=1}^M \sum_{n=1}^N [f(m, n) - g(m, n)]^2}{\sum_{m=1}^M \sum_{n=1}^N [f(m, n) - \hat{f}(m, n)]^2} \right\}, \quad (27)$$

where $f(m, n)$ and $g(m, n)$ are the original and degraded images and $\hat{f}(m, n)$ is the estimated true image. ISNR cannot be used when the true image is unknown, but it can be used to compare different methods in simulations when the true image is known.

Note that the CM cost is non-convex. Hence, the new method may converge to a local minimum instead of the global minimum of J_{CM} depending on how it is initialized. If there is no a priori information about the PSF, the adaptive filter is initialized using a 2-D spike characterized by a non-zero coefficient usually located somewhere in the central portion of the adaptive filter. If there is a priori information about the PSF, this information may aid in selection of the non-zero element in the adaptive filter. Since it was assumed that the PSF is unknown, a 2-D impulse function was used as the initial filter for the method.

Figures 3–6, in which the true images, blurred images and estimated true images are depicted in the left, middle and right column, respectively, illustrate how the method behaves as the number of gray levels increases (as the kurtosis of the true image increases). According to 1-D CMA analysis [9], increases in the source kurtosis as long as they remain sub-Gaussian ($\gamma < 3$) do not effect the location of CM cost minima, they flatten the CM surface which means that the algorithm will converge to the minima more slowly. Furthermore, increases in the source kurtosis raise the CM surface, and in turn, increase the excess CM cost. It is clear from Figs. 3–6 that performance of the new method depends on the normalized kurtosis of the true image. As the true image kurtosis increases, performance degrades, which is in agreement with the 1-D theory. However, it is impossible to pictorially verify the 1-D results given in Ref. [9] since the CM cost surface is a 25-dimensional space for a 5×5 adaptive filter.



Fig. 3. Blind deconvolution result for a binary image, no noise. True image (left); blurred image (middle); estimated true image (right), ISNR = 19.93 dB.



Fig. 4. Blind deconvolution result for a 2-bit image, no noise. True image (left); blurred image (middle); estimated true image (right), ISNR = 13.66 dB.



Fig. 5. Blind deconvolution result for a 3-bit image, no noise. True image (left); blurred image (middle); estimated true image (right), ISNR = 10.56 dB.

As stated in Section 3, the adaptive filter provides an approximate inverse of the blur at convergence. Figure 7 supports this claim by showing the magnitudes of the 32×32 -point two-dimensional discrete-time Fourier transform (DTFT) of the scatter blur and the adaptive filter at convergence for the 2-gray level case. Here, we are showing only one result because of space limitation. It is clear from Fig. 7 that the adaptive filter is an approximate inverse of the blur at convergence for this case. Similar results were obtained for the other gray levels.



Fig. 6. Blind deconvolution result for a 4-bit image, no noise. True image (left); blurred image (middle); estimated true image (right), ISNR = 6.12 dB.

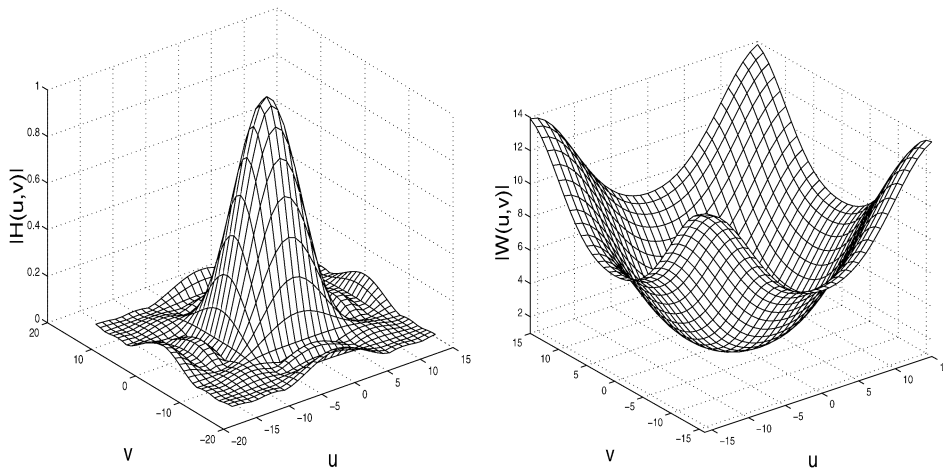


Fig. 7. The magnitudes of the 32×32 -point DTFT of 5×5 scatter blur with $\beta = 2$ (left); adaptive filter at convergence (right).

Next, performance of the method is demonstrated as a function of BSNR for a given 2-bit true image. In 1-D, if the additive noise is the only violation of the global convergence conditions, location of global CM minimum shifts towards the origin in equalizer parameter space and the minimum achievable CM cost is increased [9]. Similar events are likely to happen in 2-D. Table 2 gives ISNR results for several BSNRs from which it is obvious that performance worsens as the BSNR increases for a fixed true image kurtosis. Therefore, results again support the 1-D theory. It appears from simulation results that the new method is useful as long as the BSNR is greater than 30 dB.

5. Conclusions

A new method that is based on non-linear adaptive filtering for blind deconvolution of noisy blurred images was proposed in this paper. The new method is essentially a 2-D

Table 2
ISNR values obtained using the new method
for a 2-bit true image at several BSNRs

BSNR (dB)	ISNR (dB)
No noise	13.66
70	13.64
50	13.35
40	10.75
30	4.2

extension of the CMA. The method can be used to initialize an adaptive blind deconvolution method or it can be used by itself. An important aspect of the method is that images which were blurred by mixed-phase phase blurs can be recovered. This is due to the fact the method does not impose constraints on the phase of the blur. Another important aspect is that the method is computationally simple, which makes its implementation easy for real applications. Simulations have shown that the performance of the new method depends on the normalized kurtosis or equivalently the number of gray levels of the true image and the BSNR. The algorithm was justified via simulation. Further research is needed to improve computational aspects of the algorithm, to concretely describe its behavior in a variety of situations and to transform it into a reliable and robust technique for blind image deconvolution. To this end, the static and dynamic convergence analysis of the method near the global minimum of J_{CM} was performed in Ref. [10], according to which, given a step-size and a PSF, there is an optimum support for the adaptive filter that can be determined experimentally. Note that the new method could be implemented using an autoregressive (AR) filter instead of an FIR filter. In the AR implementation, there is no need to determine the optimum filter support experimentally if the support of the PSF is known. The AR case is fully discussed in Ref. [10].

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