A consonance-based approach to the harpsichord tuning of Domenico Scarlatti

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This paper discusses a quantitative method for the study of historical keyboard instrument tunings that is based on a measure of the perceived dissonance of the intervals in a tuning and their frequency of occurrence in the compositions of Domenico Scarlatti (1685–1757). We conclude that the total dissonance of a large volume of music is a useful tool for studies of keyboard instrument tuning in a historical musical context, although it is insufficient by itself. Its use provides significant evidence that Scarlatti used French tunings of his period during the composition of his sonatas. Use of total dissonance to optimize a 12-tone tuning for a historical body of music can produce musically valuable results, but must at present be tempered with musical judgment, in particular to prevent overspecialization of the intervals.
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INTRODUCTION

Numerous musical scales have been used over the years for Western music, with the aim of maximizing the quality of sound produced by fixed-pitch 12-note instruments such as the harpsichord. The principal musical interval in this tradition is the octave, which is defined as a factor of 2 in frequency. Other ''pure'' intervals are defined as those formed by small integer ratios; such intervals have maximum overlap of harmonic partials of these instruments and hence tend to minimize beating. Given that the octaves are to be exactly in tune, only a few of the other intervals can be pure in a single fixed tuning.

The modern preference is to make all intervals somewhat ''impure'' by using 12 equal semitones. There is then no difference between various musical keys and there are no restrictions on modulation. Unfortunately, this also removes key tonality as a significant structure in music. Three hundred years ago this was not the case. Until about 1780, the preference was to tune keyboard instruments so that commonly used intervals were purer than those less used. The resulting nonequal semitones gave a different harmonic ''color'' to each musical key. These harmonic colors were part of the musical language of the time, philosophically and practically. If one wishes to understand the musical language of early keyboard composers, the tuning in which their music was conceived and heard is important.

However, few composers documented the exact tunings used in their music. Although there is sufficient historical evidence that the period and nationality of a composer can narrow the choice considerably, there are often significant variances between historically justifiable tunings for any specific piece of music. One of us (JS) has recorded the complete harpsichord works of Domenico Scarlatti (1685–1757) in the style of his time. Scarlatti's tuning preferences are particularly uncertain, since he was born and trained in Italy, but spent most of his career in Portugal and Spain, and did all of his significant composing while clearly under strong Spanish influence. A method which might infer information concerning his tuning preferences solely from his surviving music would therefore be of value to musicians and musicologists.

This paper discusses a quantitative method that is based on a measure of the perceived consonance of the intervals in a tuning and their frequency of occurrence in the compositions of Scarlatti. Its presumption is that Scarlatti would avoid passages using intervals that were markedly out-of-tune or dissonant in his tuning (such as wolf fifths) except in passing, and would tend on average to emphasize those intervals and keys which were relatively pure. Unlike traditional approaches to this question,[1] which are based on the same assumption, but rely upon culture-dependent interval selection and classification, this method is based directly upon experimentally determined psychoacoustic properties of human hearing.

Sethares[2] introduced a parameterization of the perceptual data of Plomp and Levelt[3] that can be used to calculate the perceived dissonance (or equivalently, the perceived consonance) of an interval given the spectrum of a sound. It can be used to relate the timbre of a musical sound to a set of intervals (or scale) in which the sound can be played most consonantly. This paper reports on the results of applying this approach to investigate harpsichord tunings of Domenico Scarlatti, by finding tunings which minimize the dissonance over all intervals actually used by Scarlatti in his sonatas. The resulting tunings are then compared both theoretically and using informal listener tests to several well known historical tunings. The method should be applicable to other early keyboard composers.

I. METHODOLOGY

There are four basic steps to find the most consonant tuning for a piece (or collection of pieces) of music. These are:

(1) Specify the timbre (spectrum) of each sound.
(2) Find (or count) the number of occurrences of each interval class, and weight by their duration.
(3) Choose an initial ''guess'' for the optimization algorithm.
(4) Implement a gradient descent (or other local optimization algorithm) to find the nearest ''least dissonant'' set of intervals.

The bulk of this section describes these four steps in detail. Since the Scarlatti sonatas were composed mostly for harpsichord, a spectrum was chosen that approximates an idealized harpsichord string. The tone was assumed to contain 32 harmonic partials at frequencies

$$(w, 2w, 3w, \ldots, 32w), \tag{1}$$

where w is the fundamental. The amplitude of the partials was assumed to decrease by a factor of 0.75 (2.5 dB) per partial. Surviving historical harpsichords vary considerably in these parameters. The low strings of some have many more than 32 discernable partials, decreasing by a factor as high as 0.9, while the high strings of others display as few as 8 partials. The amplitudes of partials also have periodicities arising from the position at which the string is plucked, which varies widely, and room acoustics of the time also vary. We consider that the spectrum chosen is a reasonable approximation to the sounds of the harpsichords on which Scarlatti most likely played, in the portion of their range in which a musician is most sensitive to questions of tuning.

The Scarlatti recordings are in MIDI file format,[4] which is a widely accepted standard for encoding the finger motions of a keyboard player as a function of time. These finger motions are then transformed by a ''patch,'' a representation of the sound produced by the instrument used for the recording, to synthesize a performance similar to that heard in the recording studio. A program was written to parse these MIDI files and to collate the required information about frequency of occurrence of intervals and their duration in actual performance.

The dissonance between any two partials (pure tones) at frequencies $f1$ and $f2$, with amplitudes $v1$ and $v2$ can be calculated as

$$d(f1, f2, v1, v2) = v1 v2 (e^{-ax} - e^{-bx}), \tag{2}$$

where

$$x = 0.24 |f1 - f2| / [0.021 \min(f1, f2) + 19],$$

$$a = 3.5, \text{ and } b = 5.75. \tag{3}$$

The dissonance D( ) of a pair of notes $n1$ and $n2$ with fundamentals $w1$ and $w2$ is obtained by summing all the dissonances of all the partials to give

$$\mathbf{D}(n1, n2) = \sum_{i=0}^{31-k} \sum_{j=i+k}^{31} d((i+1)w1, (j+1)w2, 0.75^i, 0.75^j), \tag{4}$$

where $k=1$ to exclude unisons, which cannot be produced on a single manual harpsichord. This parameterization of the Plomp and Levelt psychoacoustic data was presented in Sethares,[2] where typical forms of the functions $d( )$ and D( ) are shown. The total dissonance (TD) of a musical passage of m notes is defined to be the sum of the dissonances weighted by the duration over which the intervals overlap in time, thus

$$\text{TD} = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} D(i,j) t(i,j), \tag{5}$$

where $t(i,j)$ is the total time during which notes i and j sound simultaneously. Since d is always positive, D( ) and TD are always positive as well. Although the amplitude of a single held note of a harpsichord decreases with time, it increases significantly each time a succeeding note is played due to coupling via their shared soundboard. Given the high note rates in the sonatas, the rectangular sound intensity distribution implied by (5) is a reasonable approximation to make.

We define a ''tuning'' to be a set of 11 distinct intervals between 1:1 and 2:1. Then the TD for a musical composition differs depending on the tuning chosen since the different intervals have different values of $D(i,j)$. By choosing the tuning properly, the total dissonance of the passage can be minimized, or equivalently, the consonance can be maximized. Thus the problem of choosing the tuning that maximizes consonance can be stated as an optimization problem: minimize the ''cost'' (the TD of the composition) by choice of the intervals that define the tuning. This optimization problem can be solved using a variety of techniques; we chose a gradient descent method similar to that of Sethares.[5] Indeed, the adaptive tuning method can be considered a special (instantaneous) case of the present method, which maintains a history of the piece via the $t(i,j)$ terms.

Let I_0 be the initial ''tuning vector'' containing a list of the intervals that define the tuning. A (locally) optimal I_* can be found by iterating

$$I_{k+1} = I_k - \mu \frac{d\text{TD}}{dI_k}, \tag{6}$$

until convergence, where μ is a small positive step size and k is the iteration counter. The algorithm is said to have converged when $\Delta\text{TD}/\Delta I_k$ is positive for all $\Delta I_k = \pm 0.025$ cents. (0.025 cents is the default pitch resolution of systems meeting the general Midi specification.) D( ) is a symmetric matrix with dimension equal to the number of notes of Scarlatti's harpsichords (up to 60), but since all tunings use pure octaves, the D( ) map into tuning vectors I_k and I_* of exactly 11 elements. Calculation is straightforward, although rather tedious. In most cases, the algorithm was initialized at the 12-tone equal tempered scale, that is, I_0 was a vector in which all adjacent intervals are equal to the twelfth root of 2,

TABLE I. Total dissonances relative to 12 tet for various tunings.

Historical tunings:	TD	s	Cents ($C\equiv 0$)										
12 tet	0	0	100.0	200.0	300.0	400.0	500.0	600.0	700.0	800.0	900.0	1000.0	1100.0
Bethisy	−0.4	4.1	86.8	193.2	288.9	386.3	496.3	586.6	696.6	787.0	889.7	992.6	1086.5
Rameau b	−0.5	7.1	92.5	193.2	304.8	386.3	503.4	582.2	696.6	800.0	889.7	1006.8	1082.9
Werckmeister 5	−0.6	2.6	96.1	203.9	300.0	396.1	503.9	600.0	702.0	792.2	900.0	1002.0	1098.0
d’Alembert	−0.8	4.1	86.8	193.2	290.2	386.3	496.7	586.6	696.6	787.0	889.7	993.5	1086.5
Barca	−1.0	2.4	92.2	196.7	296.1	393.5	498.0	590.2	698.4	794.1	895.1	996.1	1091.9
Marpourg	−1.3	6.1	84.3	193.2	293.8	386.3	503.4	579.5	696.6	789.1	889.7	998.6	1082.9
Werckmeister 4	1.4	5.2	82.4	196.1	294.1	392.2	498.0	588.3	694.1	784.4	890.2	1003.9	1086.3
Vallotti	−1.6	2.9	90.2	196.1	294.1	392.2	498.0	588.3	698.0	792.2	894.1	996.1	1090.2
Barca A	−1.7	2.4	92.2	200.3	296.1	397.1	498.0	593.8	702.0	794.1	898.7	998.0	1095.4
Werckmeister 3	−1.9	3.1	90.2	192.2	294.1	390.2	498.0	588.3	696.1	792.2	888.3	996.1	1092.2
Kirnberger 3	−1.9	3.4	90.2	193.2	294.1	386.3	498.0	590.2	696.6	792.2	889.7	996.1	1088.3
Corrette	−2.2	6.8	76.0	193.2	288.8	386.3	503.4	579.5	696.6	783.4	889.7	996.1	1082.9
Vallotti A	−2.5	2.9	90.2	200.0	294.1	396.1	498.0	592.2	702.0	792.2	898.0	996.1	1094.1
Chaumont	−3.3	7.7	76.0	193.2	288.8	386.3	503.4	579.5	696.6	772.6	889.7	996.1	1082.9
Rameau #	−4.0	7.1	76.0	193.2	285.6	386.3	497.9	579.5	696.6	775.4	889.7	993.2	1082.9
1/4 comma A	−5.8	10.3	76.1	193.2	310.3	386.3	503.4	579.5	696.6	772.6	889.7	1006.8	1082.9
Kirnberger 2	−6.0	4.5	90.2	203.9	294.1	386.3	498.0	590.2	702.0	792.2	895.1	996.1	1088.1
derived tunings:													
TDE	−1.6	2.2	98.0	200.0	302.0	402.1	505.5	605.3	698.1	800.0	900.2	1003.9	1104.1
TDA1	−2.3	4.6	85.6	193.4	291.4	386.3	498.0	584.8	696.8	787.5	888.7	994.9	1086.5
TDA2	−7.1	5.6	87.8	200.1	294.1	386.3	498.0	586.4	698.2	789.7	884.4	996.1	1084.5
K828 LS.27			86.3	192.4	289.7	385.2	495.8	585.8	695.8	785.9	888.9	993.0	1086.2
Minimum-TD tunings of individual sonatas:													
K1 L366			85.3	197.1	⋯	401.0	512.7	583.4	695.1	787.3	899.0	1010.7	1102.9
K6 L479			111.7	203.9	315.6	386.3	498.0	582	702.0	813.7	884.4	996.1	1088.3
K11 L352			⋯	203.9	315.7	384.6	498.0	583.2	702.0	813.7	905.8	1017.6	1088.3
K20 L375			104.8	216.5	287.2	398.9	517.6	602.8	714.6	785.2	897.0	989.1	1100.9

defined as 100 cents, about 1.059. In all cases in this paper, C is defined as 0 cents.

A tuning for which a desired composition (or collection of compositions) has smaller TD is to be preferred as far as consonance is concerned. In the context of attempting to draw historical implications, the measure of TD may provide reason for rejecting tunings (those which are overly dissonant) or reconsidering tunings (those with near-optimal values of TD). As we shall argue, such judgments cannot yet be made mechanically, but must still be tempered with musical insight. The variation in values of the TD for different tunings are quite small, less than 1‰ between musically useful tunings, and are therefore expressed in parts per thousand (‰) difference from 12-tone equal-tempered (12-tet) tuning. A difference of 1‰ is clearly audible to a trained musical ear in typical musical contexts. (For this reason, a numerical precision of nine decimal places or greater is advisable for the calculations of TD.)

Music of course does not consist solely of consonances. Baroque music is full of trills and similar features which involve overlapped seconds in real performance, and Scarlatti made heavy use of solidly overlapped seconds, deliberate dissonances, as a rhythmic device. We therefore omitted all intervals smaller than 3 semitones from the calculations of the TD, that is we used $k=3$ in (4). Thus, the consonance of large intervals occurring during periods of small-interval dissonance were still counted. We note that this had surprisingly little effect on the values of the convergent tunings; the precaution is probably unnecessary with other composers. The contrast between dissonance and consonance, sometimes useful musically, is not dealt with by the consonance method, but it is also not dealt with by that of Barnes.[1]

II. RESULTS AND DISCUSSIONS

Since harpsichords (in contrast with organs) were tuned very frequently, usually by the performer, one assumes that composers might have changed their preferred tuning over the course of their lifetime, or used more than one type of tuning depending upon the music to be played. In fact, both of these are well documented in the case of Rameau. So, we began the analysis by initializing the tuning vector I_k to the intervals of 12-tet, and found the optimum tunings I_*, those that minimize the TD for each of the 536 harpsichord sonatas individually, in the hope of discerning any such changes. A few example results are included in Table I. (Throughout, K numbers are those of the chronological Kirkpatrick catalogue,[6] L numbers those of the widely available Longo edition.) A histogram of all the tunings thus obtained is shown in Fig. 1. In this graph, the height of a bar shows for how many sonatas the optimum tuning contained a note of the pitch shown. As can be seen, for most of the 11 pitches, there were two strong preferences. Below the frequency bars is shown the location of the pure fifths (3:2, 701.955 cents) ascending and descending from C, e.g., $701.995n \bmod 1200$ for $n=-11$ to $+11$, the well-known enharmonic pitches. The minimization process for samples as small as one sonata thus for the most part ‘‘locks on’’ to the predominate non-

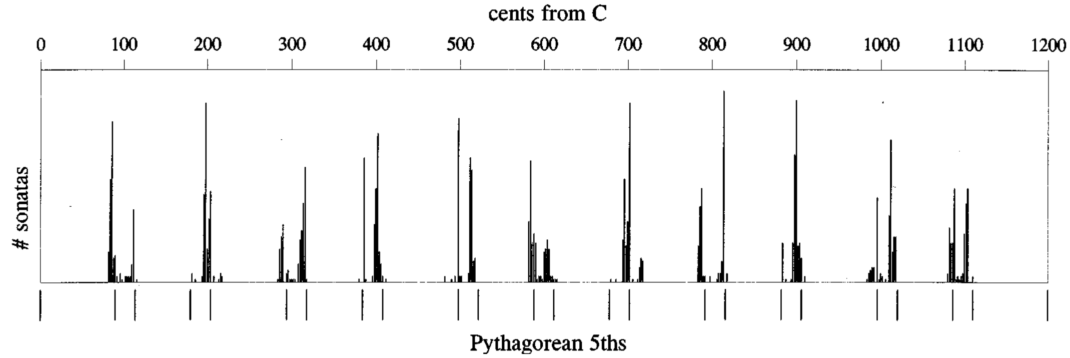

FIG. 1. Showing the relative distribution of optimal (TD minimized) tunings for each of the sonatas individually in relation to the Pythagorean cycle of pure fifths.

unison minimum in D( ), that at pure fifths.[7] This effect continued to dominate even when groups of up to ten sonatas were evaluated. Although Baroque harpsichordists would have refined the tuning of their instruments before performing suites of pieces using a consistent tonality set, just as harpsichordists do today, it is quite impractical to completely retune an instrument every 10 min (the length of a typical sonata pair with repeats and variations).

The primary formal structure of almost all of the sonatas follows two symmetries: tonalities are mirrored about a central double bar, and thematic material repeats after the double bar (although not always in exactly the same order). For example, K1 begins in D minor, progresses to A major at the double bar 14, and ends in D minor bar 31; thematically, bar 1 matches bar 14; 2–5, 22–25; 7, 17; 9, 18; 13, 31. One expects that Scarlatti's tuning(s) would have complemented and been consistent with these symmetries. Many of the single-sonata tunings found by this optimization method are not. For example, bars 9 and 18 in K1 are symmetrically designed to strongly establish the tonalities D minor and F major, respectively, but the pure D-A 5th on which bar 9 is based is inconsistent with the F-C 5th of bar 18, a very audible 15 cents smaller than pure in this tuning. By comparison, these intervals differ by only 4 cents in the Vallotti A tuning. Using optimized tunings to retune sections of music of sonata length does not, therefore, seem to be a reliable guide to the practice of Domenico Scarlatti, nor to be useful in detecting changes in tuning preferences over his oeuvre.

We then evaluated the relative TD of a number of tunings that are documented in the musical literature of Scarlatti's time. Meantone tuning, in which all fifths are equal excepting one ''wolf'' fifth G♯-E♭, was the commonest tuning at the close of the Middle Ages. It was considered to be in the key of D, and was modified steadily towards equal tempering by increasing the size of the equal fifths as time progressed. However, since only one note needs to be retuned in order to transpose any meantone tuning into the tuning for an adjacent fifth (e.g., to add or subtract one sharp or flat from the key signature), many performers did so to improve the sound of their favourite keys. The ''consonance'' (negative of TD, so that the minima are visible as peaks) relative to 12 tet for the set of all the Scarlatti sonatas is shown in Fig. 2 versus the size of the equal fifths and the position of the wolf fifth. There is a sharp minimum of TD with fifths 3.42 cents less than 12-tet when the wolf is between E♭ and B♭ or between E♭ and G♯, precisely the medieval 1/4-comma tunings in the keys of A and D. There is another broader minimum with fifths 1.8 cents larger than 12 tet, which is close to the ancient Pythagorean tuning with pure fifths. The general shape of the meantone TD of the entire keyboard oeuvre of Scarlatti is therefore in accord with historical musical practice.

Many historical harpsichord tunings have been quantified by Asselin;[8] the tunings used in this study are shown in Table I. Since the harpsichord scale has 11 degrees of freedom, it is desirable to characterize each tuning by a smaller number of musically useful parameters in order to clarify the results of the analysis. We have chosen for this purpose the mean absolute difference between the various tunings and 12 tet as a strength parameter s, i.e.,

$$s(t)=\overline{|c(k,e)-c(k,t)|}, \tag{7}$$

where $c(k,e)$ is the pitch in cents of note k from note 1 of the 12-tet scale, $c(k,t)$ the corresponding pitch of tuning t, and

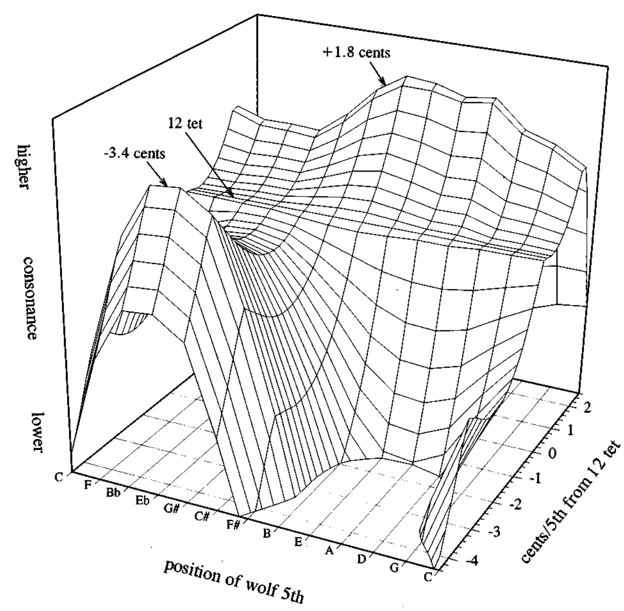

FIG. 2. Showing the relation between consonance (the negative of TD) relative to 12 tet, size of equal fifths in a meantone-type tuning and position of the unequal ''wolf'' fifth, for all the sonatas as one unit.

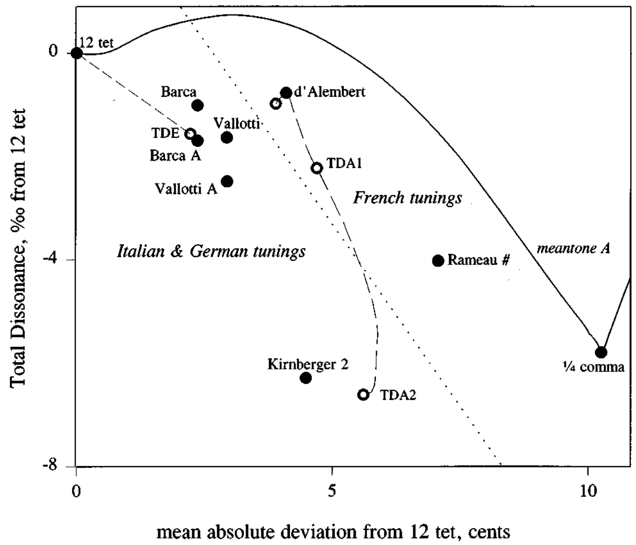

FIG. 3. Showing the total dissonance (TD) relative to the equal-tempered tuning (12 tet) of all the sonatas as one unit versus the mean absolute deviation from 12 tet for some tunings of Table I.

$$\overline{c(k,e)}=\overline{c(k,t)} \tag{8}$$

to remove the pitch scale dependence of the dissonance function d. Historically, the value of $s(t)$ has decreased with time, from 10 cents for the medieval 1/4-comma meantone tuning to essentially zero for modern piano tunings. In general, a low value of s is associated with tunings that work in a wide variety of keys, a high value with tunings placing many restrictions on modulation.

The values of the TD (5) for the recorded sonatas for these tunings are shown by solid circles in Fig. 3 versus the strength of the tuning (7). If a series of meantone-type tunings in A is constructed, with the size of the equal fifths decreasing from 12 tet (100 cents) to 96 cents, the locus of TD and s is the solid line shown. (It is the same curve as that for the wolf between E♭ and B♭ in Fig. 2.) In Fig. 3, a decrease of both the TD and s should be an improvement both in consonance and in modulatability. A decrease in the TD associated with an increase in s will however involve a choice based on musical context, since any improvement in consonance may be offset by a reduction in the range of keys in which the consonance will be useful.

In general, French tunings sought to purify the sound of major thirds, whereas Italian and German tunings were more closely derived from the fifth-based meantone. The two schools may be separated by the dotted line in Fig. 3; again the TD is in accordance with historical knowledge. Both Italian tunings in A show superior consonance to those in D, and Rameau's ''sharp'' tuning has greater consonance than that in Bb. (Modulated versions of any tuning have the same strength s.) The expectation from this figure is that Kirnberger 2 should be by far the best tuning for the sonatas, with meantone (1/4 comma) second except perhaps in some remote tonalities due to its strength. Next should be the sharp tuning of Rameau (again with possible difficulties in some tonalities), followed by Vallotti A, then Barca A. Unfortunately, other factors intervene.

A primary phrase pattern widely used in Western music, and particularly by Scarlatti in the sonatas, is a gradual increase of musical tension culminating in a musical steady state (stasis) or a release of tension (resolution). Increasing pitch, volume, rapidity, harmonic density, and harmonic dissonance are techniques of increasing musical tension. A skilled composer will use these various techniques in a mutually supporting way, in consistent patterns. If, therefore, use of a particular tuning enhances the ebb and flow of musical tension, it may be the tuning that the composer used to hear music. Since only a small proportion of potential intervals can be simultaneously in perfect tune in one tuning, it is likely that an erroneous tuning at least occasionally results in a glaring mismatch of musical shape.

The TD predictions fail badly with the tuning 2 of Kirnberger when this tension structure is taken into account; the consonances in this tuning generally fall in Scarlatti's relatively long tonal transition passages and all too frequently come to abrupt halts with unacceptably dissonant stases. For example, sonata K1 begins the second section with an A major triad ascent to an E in the treble, then repeats the figure in the bass under the sustained E. With Kirnberger 2, A-E is almost 11 cents smaller than pure, one of the most dissonant 5ths in the tuning. In both the Vallotti A and d'Alembert tunings, by comparison, A-E is a bit less than 1/4 comma smaller than pure, precisely right for an interim pause in the overall upward passage of which the A to E phrase forms a part. Beside frequently failing the tension-topology criterion, and the symmetry criteria discussed earlier, the 1/4-comma meantone tuning too often produces phrases that stay consistently out of tune for too long at a time for the ear (although obviously not long enough to affect the TD sufficiently), for example, the chromatic passages in bars 10–14 and 35–38 of K3 L378. In fact, these bars together with their symmetric pair 58–63 and 84–87 cannot be played in consistent tune with any placement of a 1/4-comma tuning wolf fifth.

However, although the tonal colours of Rameau are clearly in evidence to the modern ear, so are the consonances, which fall in the right places, and the tuning is particularly evocative in many of Scarlatti's slow plaintive melodic passages (K11, for example). The smooth matches of the Vallotti A tonal structure with those implicit in the music are very consistent, if unremarkable. The French tunings do indeed mostly have problems with dissonances in many places (the chromatic passages of K3, for example). However, the tuning of d'Alembert, despite its height on the graph, is a good overall musical match to the music, although few of its consonances seem really pure using the values of Asselin.

To quantify these judgments a bit better, a sample of 10% of the sonatas (every tenth) was used for a comparative listening test with the most promising tunings. The question asked for each of the 50 sonatas in the sample was: given that the tuning in question was the normal tuning colour of the culture, would a good composer have written this way for it? Since these tunings are rooted in history and, as a soloist, Scarlatti was free to tune his own instrument and to control his own sounds, this form of question is, we submit, appropriate to the purpose of this paper. (One would expect quite

TABLE II. Aural ranking of some tunings (see text for details).

Barca A	50
Vallotti A	46
TDA1	44
Werckmeister 3	39
Rameau #	34
Kirnberger 2	15
Meantone A	<0

different answers from musically naive listeners of the time, or of today, as well as from modern musicians unsteeped in baroque culture.) A single really unmusical dissonance, or serious mismatch between consonance shape and phrase shape rated a minus 1, more than a few awkward-sounding phrases rated a zero, while sonatas encountering no significant difficulties received $+1$. 12 tet is, of course, degenerate with respect to this question, and tunings that approach it would, in the limit, have a score of 50. Thus, a ''mild'' tuning will automatically receive a higher score than a strong tuning, which must be taken into account when comparing ratings. All the ratings were done by one person (JS) under consistent listening conditions. The results are shown in Table II. Although the rating is in the conventional form of a hypothesis to be disproven, it has a second interpretation—the fact that a tuning as strong as that of Rameau encounters so few difficulties must be considered as evidence on its own that Scarlatti had French tunings in mind as he composed. Original artists like Scarlatti often bend conventional notions to their limit, so the most likely tunings are not those with a perfect score, but rather the strongest ones that still have good scores. This suggests that the tuning of d'Alembert, descended from the earlier tuning of Rameau, is a very likely historically documented candidate for the tuning that Scarlatti used as he composed his music. We note that this is unexpected based upon historical evidence.

The historical instructions for some tunings are uncertain, even deliberately ambiguous, so modern numeric reconstructions may be slightly in error. This is almost certainly the case for the tuning of d'Alembert, which was described and redescribed in remarkably varied terms by several authors (e.g., Bethisy) of the time. We again applied the gradient algorithm with several cost functions to successively reduce the TD in small steps for the set of all the sonatas, beginning with this tuning (instead of initializing with 12 tet), with the hope that this might correct minor errors in what is basically a good tuning. We found that the routes of the optimizations as well as the final points were affected very little by step size below 0.5 cent, and also affected little by the number of harmonics in the timbre above 8, so computation time was reasonable (a weekend on a PC for the longest). Two routes the algorithm took are shown by dashed lines in Fig. 3. The longest (right) curve shows the route when the only criteria for the change in I was lower TD, the shortest (left) when I was optimized for lower s and lower TD simultaneously. The first minimization proceeded well beyond the best-sounding point along the path, ending up at a tuning (TDA2 in Fig. 3) that made the commonest intervals perfectly consonant but far too many lesser-used musically important ones unacceptably dissonant (for example, the repeated high D-A fifths of K1, 17 cents flat). A selection of the tunings along this path were listened to, and the musically most promising one (TDA1) chosen for a full listening test. It is an excellent tuning for the oeuvre. In fact, when all the 536 recorded sonatas were subjected to the criteria used for Table II with it, not a single sonata was rated -1, and only about 5% rated a 0.

Furthermore, if this small-step optimization from the d'Alembert tuning is applied individually to the few sonatas where the TDA1 tuning has residual difficulties, a similar behavior is observed. At first, the sound improves, and then, with further iteration, the tuning becomes ''overspecialized.'' For example, the 5ths ending many phrases of K328, and the chords closing each half, are a bit more discordant with TDA1 than one would wish, although consistently so. Applying the small-step refinement procedure for just this sonata produces the tuning included in Table I—the 5ths and chords all improve in consonance compared to TDA1, without changing the sound of the rest of the sonata adversely or changing the basic colour of the tuning. This is in accordance with historical practise, where a basic tuning would be ''touched up'' for a while to play a group of pieces that benefited from it (as opposed to the minimum-TD tunings which vary too much between sonatas to be practical).

We also applied this small-step optimization beginning with other tunings, to the set of all sonatas and to some individual sonatas. The qualitative pattern observed with the d'Alembert tuning—that the overall musical quality of the tuning first improved, and then declined as the TD approached its minimum—was also observed with most other tunings. However, none improved to anything like the same extent nor with the same consistency as did the d'Alembert tuning.

III. CONCLUSIONS

In summary, the total dissonance of a large volume of music is a useful tool for studies of 12-tone keyboard instrument tuning in a historical musical context, although it is insufficient by itself. Its use provides significant evidence that Domenico Scarlatti used the French tunings of his period during the period of composition of his sonatas. Use of total dissonance to optimize a 12-tone tuning for a historical body of music can produce musically valuable results, but must at present be tempered with musical judgment, in particular to prevent overspecialization of the intervals.

[1] J. Barnes, ''Bach's keyboard temperament; Internal evidence from the Well-Tempered Clavier,'' Early Music **7**, 236–249 (1979).

[2] W. A. Sethares, ''Local consonance and the relationship between tuning and timbre,'' J. Acoust. Soc. Am. **94**, 1218–1228 (1993).

[3] R. Plomp and W. J. M. Levelt, ''Tonal consonance and critical bandwidth,'' J. Acoust. Soc. Am. **38**, 548–560 (1965).

[4] The files are currently available on the Internet at ftp://ftp.cs.ruu.nl/pub/MIDI/SONGS/CLASSICAL/SCARLATTI/.

[5] W. A. Sethares, ''Adaptive tunings for musical scales,'' J. Acoust. Soc. Am. **96**, 10–18 (1994).

[6] R. Kirkpatrick, *Domenico Scarlatti* (Princeton U.P., Princeton, NJ, 1953).

[7] See Fig. 3 in Sethares (Ref. 2).

[8] P-Y. Asselin, *Musique et tempérament* (Éditions Costallat, Paris, 1985).