ACS : 867 PROD.TYPE: TYP ED:CHANDRA ELIZABETH
pp.1–22 (col.fig.: NIL) 3B2 PAGN: VIMALA SCAN: GIRISH

INTERNATIONAL JOURNAL OF ADAPTIVE CONTROL AND SIGNAL PROCESSING
Int. J. Adapt. Control Signal Process. 2005; **19**:000–000
Published online in Wiley InterScience (www.interscience.wiley.com) DOI:10.1002/acs.867

Recursive blind image deconvolution via dispersion minimization

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SUMMARY

This paper presents an adaptive autoregressive (AR) approach to the blind image deconvolution problem which has several advantages over standard adaptive FIR filters. There is no need to figure out the optimum filter support when using an AR deconvolution filter because it is the same as the support of the blur. Thus there is no distortion introduced by the finite support of the FIR filter. While an FIR filter provides an approximate inverse to the blur at convergence, the AR filter converges to an approximation of the blur itself. Hence, the method can be used for blur identification. Simulations suggest that convergence of the adaptive AR filter coefficients occur rapidly and the improvement in signal-to-noise ratios are higher than in the FIR case for a given blur (and with the same step-size for the adaptive algorithms). When the adaptive AR method is derived naively to minimize the dispersion, it requires a recursion within a recursion which is computationally complex. We propose a simplification that removes the inner recursion, and prove conditions under which this simplification is valid when dealing with binary images. Simulations are used to show that the method may also be applied to certain multi-valued images as well. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: image restoration; blind image deconvolution; blur identification; constant modulus algorithm; recursive adaptive filtering; local stability analysis

1. INTRODUCTION

A recorded image is usually a degraded version of the original because physical imaging systems are not perfect. Blur and observation noise are the most common degradations seen in recorded images, and often are unavoidable. The central problem in the field of image restoration is to reconstruct an unobservable *true image* from an observed *degraded image*.

If the blur, which is often called the point spread function (PSF) in the literature, is assumed to be a linear shift invariant (LSI) system, an observed image can be written (ignoring

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Received 6 October 2004
Revised 15 October 2004
Accepted 22 November 2004

observation noise) as the two-dimensional (2-D) convolution of the true image with the blur. Restoration of the true image in the case of a known blur has been studied extensively and given rise to a variety of solutions [1–4]. However, the blur is unknown in many practical cases. Hence, restoration of the true image must be performed from the degraded image alone, and this is called *blind image restoration*.

A modern comprehensive survey of existing blind image deconvolution methods can be found in the papers by Kundur and Hatzinakos [5, 6], according to which blind image deconvolution methods can be divided into two major groups: (i) those which estimate the PSF *a priori* independent of the true image so as to use it later with one of the linear image restoration methods, and (ii) those which estimate the PSF and the true image simultaneously. Algorithms belonging to the first class tend to be computationally simple, but they are limited to situations in which the PSF has a special parametric form, and the true image has certain features. Algorithms belonging to the second class, which are usually computationally more complex, must be used for more general situations. More recently, recursive schemes such as those in References [7, 8] have been introduced.

A computationally simple blind image deconvolution method that is applicable to minimum or mixed phase blurs was presented and analysed in Reference [9]. The method is essentially a 2-D version of the constant modulus algorithm (CMA) [10, 11] that is commonly used in the field of communications for blind equalization. CMA is applicable whenever the unknown input arises from a 'finite alphabet'. Since the pixels in a digitized image are drawn from a finite alphabet (often 256 levels, though sometimes as few as two[§]), the CM cost may also be useful in the deblurring and denoising of images. The reader is referred to Reference [12] and the references therein for a detailed introduction to the CMA and its analysis in the context of one-dimensional (1-D) adaptive equalization.

A 2-D version of the FIR CMA was introduced in Reference [13]. The present paper provides an analogous method that uses an adaptive 2-D autoregressive (AR) filter for deconvolution. This has several important advantages. First, analysis of the FIR implementation has shown that given a step-size and a PSF, there is an optimum support for the FIR filter that must be determined experimentally. There is no need to figure out the optimum support when using an AR deconvolution filter because the optimum support is the same as the support of the blur. Second, the FIR filter provides an approximate inverse to the blur at convergence while the AR filter converges to an approximation of the blur itself.

In 1-D, implementing an adaptive algorithm is not possible for a non-causal channel without introducing an appropriate delay. This causality issue does not impose a constraint for the blind image deconvolution problem since the observed degraded image can be used as an initial restored image. For notational simplicity, this paper focuses on 2-D AR filters and FIR blurs with spatially causal supports.[¶] The results can easily be extended to the non-causal case with suitable changes in the notation.

One way to understand the behaviour and performance of adaptive algorithms is by analysing the convergence. A static convergence analysis consists of characterizing the positions of stationary (minimum) points of the cost function, while a dynamic analysis investigates the stability, convergence and consistency of the adaptive filter coefficients.

[§]Many fax machines, laser printers and news-print use 2-level quantization.
[¶]A 2-D filter $h(m,n)$ is spatially causal if $h(m,n) = 0$ for $m, n < 0$.

In the ideal case, when there is no noise, a global minimum of the cost function occurs when the deconvolution filter is the inverse of the PSF, which is the *desired solution*. This paper demonstrates a sufficient condition on the PSF under which the recursive realization converges to the desired solution for a binary image. As will be seen, the presence of regressor filtering in the gradient makes the algorithm computationally costly. It is natural, then, to consider an algorithm that is simplified by removing the regressor filtering, and we perform a local stability analysis of this simplified algorithm. Conditions on the PSF under which this simplification is valid are explicitly derived. Because exponential stability of the linearized dynamical system to a given stationary point is a sufficient condition for local stability of the non-ideal noisy adaptive system to a region about that stationary point [14], this paper frames the convergence analysis by demonstrating the exponential stability of the linearized system. Thus, although the analysis ignores the observation noise, the results are robust to the presence of (suitably small) noises.

The paper is organized as follows. The blind image deconvolution problem is formulated for spatially causal blurs in Section 2. A statistically optimum fixed AR filter, which minimizes the mean square error between the true image and the restored image, can be designed when the autocorrelation function of the true image and the cross-correlation between the true image and the degraded image are known. Design of this filter, which is the subject of Section 3, will be called *supervised linear recursive image deconvolution* since the true image is assumed known. The recursive blind algorithm is derived in Section 4 in detail. Local stability of the simplified algorithm is presented in Section 5. Experimental results are provided in Section 6. Section 7 concludes the paper.

2. PROBLEM FORMULATION

A model that describes the relationship between the unobservable true image and the observed degraded image is required by all blind image deconvolution algorithms. In general, blurs are assumed to be linear, though they may be shift-invariant or shift-variant. Similarly, the observation noise may be modelled as multiplicative or additive. This paper assumes a shift-invariant blur and additive observation noise. Hence, the observed $M \times N$ degraded image $g(m,n)$ for $m = 0, \ldots, M-1$, $n = 0, \ldots, N-1$ is given by

$$g(m,n) = f(m,n) * h(m,n) + v(m,n) \tag{1}$$

$$g(m,n) = \sum_{k=0}^{A-1} \sum_{l=0}^{B-1} h(k,l) f(m-k, n-l) + v(m,n) \tag{2}$$

where $h(0,0) = 1$, $f(m,n)$, $h(m,n)$, $v(m,n)$ and $[0, A-1] \times [0, B-1]$ represent the (m,n)th pixel of the true image, the PSF of the degrading system, additive noise that is independent of the true image, and the support of the PSF, respectively. The linear image degradation model is depicted in Figure 1.

In blind image restoration, the PSF $h(m,n)$ is unknown. Therefore, the true image $f(m,n)$ must be estimated directly from the degraded image $g(m,n)$. While the values of the pixels of the true image are unknown, certain statistical properties are known; typically pixel values must be one of a small number of possibilities. As shown in References [9, 13], ambiguities in both gain and delay are inherent to blind image deconvolution, i.e. scaling the true image pixel values by α

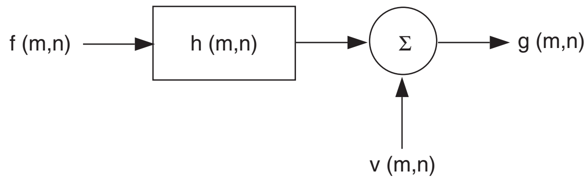

Figure 1. Linear image degradation model.

and the PSF coefficients by α^{-1} simultaneously, and advancing the true image by an integer-valued vector while delaying the PSF by the same vector do not change the observed image, where α is a real fixed non-zero gain. Keeping these ambiguities in mind, the blind image deconvolution problem can be stated more precisely as follows: Obtain an estimate of the form $\hat{f}(m,n) \approx \alpha f(m-m_0, n-n_0)$ for some real $\alpha \neq 0$ and for some integers m_0, n_0 when only the observed image $g(m,n)$ is measurable. Both the true image $f(m,n)$ and the PSF $h(m,n)$ are assumed unknown.

For the rest of the paper, pixel values of the true image are assumed odd integer-valued, i.e. pixel values may be $\pm 1, \pm 2, \ldots, \pm L-1$, where L is the number of grey levels in the true image, unless otherwise stated. Many real images are 8-bit having 256 grey levels between 0 and 255. These images can be transformed to have odd-integer-valued grey levels by thresholding based on the probability density function of the image.

3. SUPERVISED LINEAR RECURSIVE IMAGE DECONVOLUTION

Consider the general linear recursive image deconvolution problem shown in Figure 2(a), in which the goal is to estimate the true image $f(m,n)$ by designing a statistically optimum fixed filter $w(m,n)$ that minimizes the mean square error (MSE) between the true image $f(m,n)$ and the restored image $\hat{f}(m,n)$. It is well known that design of $w(m,n)$ requires that the autocorrelation function of the true image and the cross-correlation function between the true image and the degraded image be available. Suppose that this information is available. Later sections show how to roughly achieve the same goal even if this information is unavailable.

Derivation of the optimum filter in the spatial domain using Figure 2(a) is tedious. Derivation becomes straightforward in the 2-D Z-domain using the equivalent system depicted in Figure 2(b). In the following, all signals in Figure 2 will be assumed stationary. Note that the MSE can be written as

$$\begin{aligned} \text{MSE} &:= E[(f(m,n) - \hat{f}(m,n))^2] \\ &= r_{ff}(0,0) + r_{\hat{f}\hat{f}}(0,0) - 2r_{f\hat{f}}(0,0) \end{aligned} \tag{3}$$

where $r_{ff}(k_1,k_2)$, $r_{\hat{f}\hat{f}}(k_1,k_2)$ and $r_{f\hat{f}}(k_1,k_2)$ are the autocorrelation functions of $f(m,n)$, $\hat{f}(m,n)$ and the cross-correlation function between $f(m,n)$ and $\hat{f}(m,n)$ which are given by

$$r_{ff}(k_1,k_2) := E[f(m,n)f(m+k_1, n+k_2)] \tag{4}$$

$$r_{\hat{f}\hat{f}}(k_1,k_2) := E[\hat{f}(m,n)\hat{f}(m+k_1, n+k_2)] \tag{5}$$

$$r_{f\hat{f}}(k_1,k_2) := E[f(m,n)\hat{f}(m+k_1, n+k_2)] \tag{6}$$

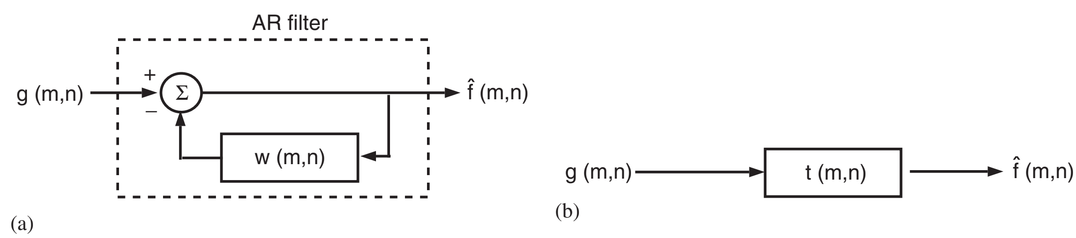

Figure 2. Supervised linear recursive image deconvolution: (a) unsimplified system; and (b) simplified equivalent system.

for $-\infty \leqslant k_1 \leqslant \infty$, $-\infty \leqslant k_2 \leqslant \infty$. Recall that the 2-D Z-transform of a correlation function $r_{uv}(k_1, k_2)$ is the corresponding power spectrum $S_{uv}(z_1, z_2)$ given by

$$S_{uv}(z_1, z_2) = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} r_{uv}(k_1, k_2) z_1^{-k_1} z_2^{-k_2} \tag{7}$$

The 2-D inverse Z-transform gives

$$r_{uv}(k_1, k_2) = \frac{1}{(2\pi j)^2} \oint_{C_1} \oint_{C_2} S_{uv}(z_1, z_2) z_1^{k_1-1} z_2^{k_2-1} \, \mathrm{d}z_1 \, \mathrm{d}z_2 \tag{8}$$

where both C_1 and C_2 are clockwise closed contours in the region of convergence of $S_{uv}(z_1, z_2)$. The MSE can be written in the 2-D Z-domain by substituting Equation (8) in Equation (3) with $k_1 = k_2 = 0$ which gives

$$\mathrm{MSE} = r_{ff}(0,0) + \frac{1}{(2\pi j)^2} \oint_{C_1} \oint_{C_2} [S_{\hat{f}\hat{f}}(z_1, z_2) - 2S_{f\hat{f}}(z_1, z_2)] z_1^{-1} z_2^{-1} \, \mathrm{d}z_1 \, \mathrm{d}z_2 \tag{9}$$

where $S_{\hat{f}\hat{f}}(z_1, z_2)$ is the power spectrum of $\hat{f}(m,n)$ and $S_{f\hat{f}}(z_1, z_2)$ is the cross-power spectrum between $f(m,n)$ and $\hat{f}(m,n)$. The power spectral relationships in Figure 2(b) can be found readily (see Reference [15]) which are

$$S_{\hat{f}\hat{f}}(z_1, z_2) = T(z_1, z_2) T(z_1^{-1}, z_2^{-1}) S_{gg}(z_1, z_2) \tag{10}$$

$$S_{f\hat{f}}(z_1, z_2) = T(z_1, z_2) S_{fg}(z_1, z_2) \tag{11}$$

where $T(z_1, z_2)$ is the 2-D Z-transform of $t(m,n)$, $S_{gg}(z_1, z_2)$ is the power spectrum of $g(m,n)$ and $S_{fg}(z_1, z_2)$ is the cross-power spectrum between $f(m,n)$ and $g(m,n)$. The MSE can be written in terms of $T(z_1, z_2)$ by using Equations (10) and (11) in Equation (9) from which the optimum $T(z_1, z_2)$ which results in the minimum MSE is given by

$$T^*(z_1, z_2) = \frac{S_{fg}(z_1, z_2)}{S_{gg}(z_1, z_2)} \tag{12}$$

The optimum $W(z_1, z_2)$ (where $W(z_1, z_2)$ is the 2-D Z-transform of $w(m, n)$) is given by

$$W^*(z_1, z_2) = \frac{1}{T^*(z_1, z_2)} - 1 \tag{13}$$

The optimum $w(m, n)$ is then obtained by taking the inverse 2-D Z-transform of $W^*(z_1, z_2)$].

4. INTRODUCTION TO THE CM COST

Even though traditional uses of the CM cost have all been 1-D, the CM cost can be extended for use in 2-D. The CM cost term was introduced for blind equalization of communication signals over dispersive channels by Godard [10] and Treichler and Agee [11]. This section generalizes the CM cost for use in 2-D by reformulating the cost for a real-valued zero-mean true image $f(m, n)$ and a real-valued PSF $h(m, n)$. It is assumed that each grey level of the true image is equally likely.[‖] The CM cost is given by

$$J_{\mathrm{CM}} \coloneqq E[(\hat{f}^2(m, n) - \gamma)^2] \tag{14}$$

$$J_{\mathrm{CM}} \coloneqq E[\hat{f}^4(m, n)] - 2\gamma E[\hat{f}^2(m, n)] + \gamma^2 \tag{15}$$

$$J_{\mathrm{CM}} \coloneqq E[\hat{f}^4(m, n)] - 2\sigma_f^2 \kappa_f E[\hat{f}^2(m, n)] + \sigma_f^4 \kappa_f^2 \tag{16}$$

where γ and κ_f are the dispersion constant and normalized kurtosis of the true image defined by

$$\kappa_f \coloneqq \frac{E[f^4(m, n)]}{(E[f^2(m, n)])^2} \tag{17}$$

$$\gamma \coloneqq \frac{E[f^4(m, n)]}{E[f^2(m, n)]} \tag{18}$$

Note that $\gamma = \sigma_f^2 \kappa_f$. It is evident from (14) that the CM cost penalizes the deviations (or dispersion) of $\hat{f}^2(m, n)$ from the constant γ, which is why it is sometimes called *dispersion* minimization. Plotting the CM cost versus the adaptive filter parameters results in a surface called the *CM cost surface*. The method of recursive blind image deconvolution via dispersion minimization attempts to estimate the true image by starting at some location on the surface and following the trajectory of steepest descent.

5. RECURSIVE BLIND IMAGE DECONVOLUTION VIA DISPERSION MINIMIZATION

In a blind image deconvolution setting, the supervised linear recursive image deconvolution method explained in Section 3 is inapplicable because the true image $f(m, n)$ is unknown. As in

‖ A suitable preprocessing of the true image such as histogram equalization may be required to satisfy this condition.

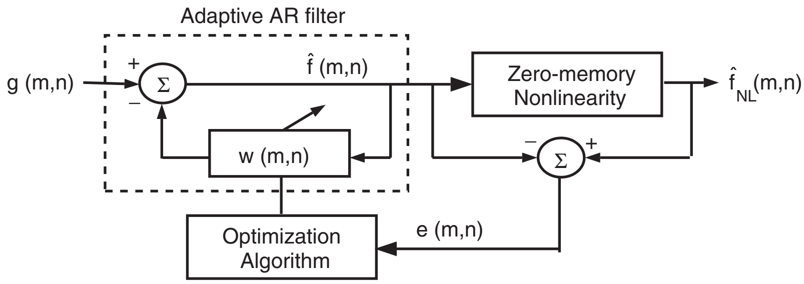

Figure 3. Recursive blind image deconvolution via dispersion minimization method.

recursive adaptive equalization, one possibility is to attempt to minimize the dispersion of $\hat{f}(m,n)$ using the CM cost. Figure 3 depicts the recursive blind image deconvolution method, where the degraded image $g(m,n)$ is applied to an adaptive AR filter whose purpose is to estimate the true image $f(m,n)$. Since the true image is unknown, a desired image (the true image in the ideal case) must be generated artificially from the estimated true image $\hat{f}(m,n)$. The function of the zero-memory non-linearity (the rightmost term in Figure 3) is to generate an 'artificial' image $\hat{f}_{\mathrm{NL}}(m,n)$ so that an error term $e(m,n) := \hat{f}_{\mathrm{NL}}(m,n) - \hat{f}(m,n)$ that drives the recursive algorithm can be obtained. The zero-memory non-linearity is chosen such that the error term $e(m,n)$ corresponds to the negative gradient of J_{CM}.

Transforming the 2-D signals and filters to the corresponding 1-D signals and filters using appropriate index mappings is useful to simplify the derivation of the recursive algorithm. A 2-D filter $w(m,n)$ with support $[0, A-1] \times [0, B-1]$ can be transformed to a 1-D filter $w(k)$ by the 'lexicographic ordering' $T_1 : R_2 \to R_1$ such that $k = mB + n$, where

$$R_2 = \{(m,n) \,|\, 0 \leqslant m \leqslant A-1, \quad 0 \leqslant n \leqslant B-1\} \tag{19}$$

$$R_1 = \{k \,|\, 0 \leqslant k \leqslant AB-1\} \tag{20}$$

Similarly, a 2-D signal $f(m,n)$ can be transformed to a 1-D signal $f(k)$ by the 'local lexicographical ordering of support $[0, A-1] \times [0, B-1]$' $T_2 : R_2 \to R_1$ such that

$$R_2 = \{(r,s) \,|\, m - A \leqslant r \leqslant m, \; n - B \leqslant s \leqslant n\} \tag{21}$$

$$R_1 = \{t \,|\, k - AB + 1 \leqslant t \leqslant k\} \tag{22}$$

where $0 \leqslant m \leqslant M-1$, $0 \leqslant n \leqslant N-1$, and $k = T_2(m,n)$ is a suitable function of (m,n). The output of the AR filter at the jth iteration $\hat{f}_j(m,n)$ is an estimate of the true image given by

$$\hat{f}_j(m,n) = g(m,n) - \sum_{r=0}^{A-1} \sum_{s=0}^{B-1} w_j(r,s) \hat{f}_j(m-r, n-s), \quad (r,s) \neq (0,0) \tag{23}$$

This estimate can be rewritten using T_1 and T_2 as

$$\hat{f}_j(k) = g(k) - \sum_{i=1}^{AB-1} w_j(i) \hat{f}_j(k-i) \tag{24}$$

where $g(k)$, $\hat{f}_j(k)$ and $w_j(i)$ are the 1-D representation of the degraded image, the output of the AR filter and the adaptive filter coefficients at the jth iteration resulting from applying the index mappings to their 2-D counterparts.**

The adaptive AR filter should be close to the global minimum of the CM cost J_{CM} to produce a reliable estimate at its output. Initially, the adaptive filter is far from being at a local or global minimum of J_{CM}. Hence, the estimate $\hat{f}_j(k)$ is not reliable enough, though it may be used in an adaptive scheme to obtain a better estimate for the next pixel by minimizing the CM (dispersion) cost. Gradient descent (GD) methods are generally used to solve for CM estimators because closed-form expressions do not usually exist. Since exact GD requires statistical knowledge of the degraded image that is unavailable in real applications, stochastic GD method are utilized. The general form of the recursive stochastic GD algorithm for minimizing the CM cost is

$$w_{j+1}(l) = w_j(l) - \mu \frac{dJ_{CM}}{dw_j(l)}, \quad l = 1, \ldots, AB - 1 \tag{25}$$

where μ is a small positive step-size. Because it is not possible to minimize an expected value directly, the method uses an instantaneous estimate J of J_{CM} given by

$$J := \tfrac{1}{4}(\hat{f}_j^2(k) - \gamma)^2 \tag{26}$$

The constant factor $\frac{1}{4}$ in Equation (26) is used to cancel a factor 4 that appears in the formula for $dJ/dw_j(l)$. Therefore, for the kth pixel coefficients of the adaptive filter are updated according to

$$\begin{aligned} w_{j+1}(l) &= w_j(l) - \mu \frac{dJ}{dw_j(l)} \\ &= w_j(l) - \mu \frac{dJ}{d\hat{f}_j(k)} \frac{d\hat{f}_j(k)}{dw_j(l)} \end{aligned} \tag{27}$$

The first derivative of Equation (27) is

$$\frac{dJ}{d\hat{f}_j(k)} = (\hat{f}_j^2(k) - \gamma)\hat{f}_j(k)$$

It is not possible to write a closed-form expression for the second derivative in Equation (27), but the derivative can be calculated iteratively using regressor filtering. To derive this term, note that $\hat{f}_j(k)$ can be written as

$$\hat{f}_j(k) = g(k) - \sum_{i=1}^{AB-1} w_j(i)\hat{f}_j(k - i) \tag{28}$$

$$\hat{f}_j(k) = g(k) - w_j(l)\hat{f}_j(k - l) - \sum_{i=1}^{AB-1} w_j(i)\hat{f}_j(k - i), \quad i \neq l \tag{29}$$

Taking the derivative of both sides of Equation (29) with respect to $w_j(l)$ gives

$$\frac{d\hat{f}_j(k)}{dw_j(l)} = -\hat{f}_j(k - l) - \sum_{i=1}^{AB-1} w_j(i)\frac{d\hat{f}_j(k - i)}{dw_j(l)}, \quad i \neq l \tag{30}$$

** In the 1-D representation, note that j is the time iteration variable, while k is the spatial position.

Let

$$\varphi_{j,l}(k) := -\frac{\mathrm{d}\hat{f}_j(k)}{\mathrm{d}w_j(l)} \tag{31}$$

Then, Equation (30) can be written in terms of $\varphi_{j,l}(k)$ as

$$\varphi_{j,l}(k) = \hat{f}_j(k-l) - \sum_{i=1}^{AB-1} w_j(i)\varphi_{j,l}(k-i), \quad i \neq l \tag{32}$$

Substituting Equation (31) in Equation (27) results in

$$w_{j+1}(l) = w_j(l) + \mu(\hat{f}_j^2(k) - \gamma)\hat{f}_j(k)\varphi_{j,l}(k) \tag{33}$$

This can be vectorized as

$$\mathbf{w}_{j+1} = \mathbf{w}_j + \mu(\hat{f}_j^2(k) - \gamma)\hat{f}_j(k)\boldsymbol{\varphi}_j(k) \tag{34}$$

where $\mathbf{w}_j$ and $\boldsymbol{\varphi}_j(k)$ are the adaptive filter coefficients vector and the regressor filter vector for the kth position given by

$$\mathbf{w}_j := [w_j(1), w_j(2), \ldots, w_j(AB-1)]^{\mathrm{T}} \tag{35}$$

$$\boldsymbol{\varphi}_j(k) := [\varphi_{j,1}(k), \varphi_{j,2}(k), \ldots, \varphi_{j,AB-1}(k)]^{\mathrm{T}} \tag{36}$$

Regressor filtering defined in Equation (32) makes implementation of the recursive algorithm costly. A simplified algorithm that bypasses the regressor filtering would be preferred. An approximate gradient for the recursive case uses the currently available data vector in place of the regressor filtered version, that is,

$$\boldsymbol{\varphi}_j(k) = [\hat{f}_j(k-1), \hat{f}_j(k-2), \ldots, \hat{f}_j(k-AB+1)]^{\mathrm{T}} \tag{37}$$

Equations (29), (34) and (37) constitute the *recursive blind image deconvolution via dispersion minimization*. Each iteration corresponds to processing a pixel of the observed degraded image $g(k)$. The output of the adaptive AR filter is an estimate of the true image $f(k)$, and the coefficients $w(k)$ provide an estimate of the PSF $h(m,n)$ at convergence.

The simplified recursive algorithm is not a stochastic gradient descent algorithm because of the removal of the regressor filtering. Consequently, it is important to study its behaviour to find conditions on the PSF under which the algorithm converges to a desirable solution. Equivalently, it is required to find a sufficient condition on the PSF such that the regressor filtering defined in Equation (32) can be omitted, i.e. $\varphi_{j,l}(k)$ can be approximated by $\hat{f}_j(k-l)$. Derivation of a sufficient condition is discussed next.

6. LOCAL STABILITY ANALYSIS

This section finds conditions on the PSF that ensure local stability of the simplified algorithm for a binary images. The approach used here is similar in spirit to that in Reference [16]. The analysis is based on determining a state-variable equation for the algorithm, linearizing the state-variable equations about a desired solution, and finding a sufficient condition on the PSF such that the linearized system is exponentially stable to the origin. This 'strong' form of stability then guarantees robustness to suitable disturbances such as observation noise.

In the absence of observation noise $v(k)$, the true image $f(k)$ can be perfectly restored by setting $W(z) = H(z) - 1$, where $W(z)$ and $H(z)$ are the 1-D Z-transforms of $w(k)$ and $h(k)$ which result from applying the mapping T_1 to $w(m,n)$ and $h(m,n)$. Hence, $H(z) - 1$ achieves the global minimum of the J_{CM}, and will be used as the desired solution. At the jth iteration, the estimation error vector $\mathbf{X}_{1,j}(k)$ defined as

$$\mathbf{X}_{1,j}(k) := [f(k-1) - \hat{f}_j(k-1), \ldots, f(k-AB+1) - \hat{f}_j(k-AB+1)]^{\mathrm{T}} \tag{38}$$

and the coefficient errors vector $\mathbf{X}_{2,j}$

$$\mathbf{X}_{2,j} := [h(1) - w_j(1), \ldots, h(AB-1) - w_j(AB-1)]^{\mathrm{T}} \tag{39}$$

will be used as state-variables. The reason for choosing $X_{1,j}(k)$ and $X_{2,j}$ is that when the adaptive filter satisfies $W(z) = H(z) - 1$, then both state vectors are equal to zero, resulting in perfect image restoration. For a binary image, there is sufficient condition on the PSF such that algorithm (34) with $\boldsymbol{\varphi}_j(k)$ given as in (37) is locally stable to $W(z) = H(z) - 1$. This result will be given in Theorem 2. Some definitions and results available from the 1-D recursive adaptive filter theory are needed to fully understand what the theorem means and its proof. Background information can be found in Reference [17].

Definition 1
A rational transfer function $G(\mathrm{e}^{\mathrm{j}\omega})$ with real coefficients is 'positive real' (PR) if

$$\mathrm{Re}[G(\mathrm{e}^{\mathrm{j}\omega})] \geqslant 0 \quad \forall \omega \in (-\pi, \pi] \tag{40}$$

A transfer function is 'strictly positive real' (SPR) if

$$\mathrm{Re}[G(\mathrm{e}^{\mathrm{j}\omega})] > 0 \quad \forall \omega \in (-\pi, \pi] \tag{41}$$

Lemma 1
If a rational transfer function $G(\mathrm{e}^{\mathrm{j}\omega})$ is SPR, so its inverse $1/G(\mathrm{e}^{\mathrm{j}\omega})$. (A proof is found in Reference [17].)

Definition 2 (*persistent excitation*)
Let the notation $R > 0$ ($R \geqslant 0$) mean a symmetric matrix R is positive definite (positive semi-definite). Similarly, let the notation $R_1 > R_2$ ($R_1 \geqslant R_2$) mean $R_1 - R_2$ is positive definite (positive semi-definite) for symmetric matrices R_1, R_2. Consider now a scalar sequence $\{u(\cdot)\}$ and build a K-element vector

$$\mathbf{u}(k) := [u(k), u(k-1), \ldots, u(k-K+1)]^{\mathrm{T}} \tag{42}$$

Then, the sequence $\{u(\cdot)\}$ is said to be 'persistently exciting' (PE) if there exists some integer L, and positive constants a, b such that for all k

$$0 < aI \leqslant \sum_{i=k}^{k+L} \mathbf{u}(i)\mathbf{u}^{\mathrm{T}}(i) \leqslant bI < \infty \tag{43}$$

where I is the identity matrix. If $\{u(\cdot)\}$ is a stationary stochastic sequence, (43) can be simplified to

$$E[\mathbf{u}(k)\mathbf{u}^{\mathrm{T}}(k)] > 0 \tag{44}$$

Definition 3 (*exponential stability*)
The state variable equations

$$\mathbf{x}(n+1) = A(n)\mathbf{x}(n) \tag{45}$$

are said to be 'exponentially stable to the origin' if for any bounded initial condition $\|\mathbf{x}(n_0)\| < \infty$ with arbitrary n_0 the resulting state vector sequence $\{\mathbf{x}(\cdot)\}$ satisfies

$$\|\mathbf{x}(n)\| \leqslant \beta\alpha^{n-n_0}\|\mathbf{x}(n_0)\| \quad \forall n \geqslant n_0 \tag{46}$$

where β is some fixed constant and $0 \leqslant \alpha \leqslant 1$.

Theorem 1 (*the Hyperstability theorem*)
Consider the closed-loop system depicted in Figure 4 with input $u(k)$ and output $y(k)$ satisfying

$$\sum_{i=0}^{N} u(i)y(i) \leqslant K^2 \tag{47}$$

where K is a constant independent of N. Then, for all initial conditions both the input and output are exponentially stable to the origin if and only if $G(\mathrm{e}^{\mathrm{j}\omega})$ is SPR.

The local stability analysis of the recursive algorithm for a binary true image is based on the Hyperstability theory of Popov [18], which encompasses a particular class of non-linear feedback systems, including those which arise in adaptive IIR filtering. Pioneered by Landau [19], the Hyperstability theory has been a important tool for analysis of the adaptive IIR filtering systems [20, 21]. The main result whose proof is given in Appendix A can be stated now.

Theorem 2
For a binary image, in the absence of observation noise $v(k)$, a sufficient condition for local stability of the simplified recursive algorithm (34) with $\boldsymbol{\varphi}_j(k)$ given as in (37) to $W(z) = H(z) - 1$ is that the lexicographically ordered PSF $h(k)$ satisfies an SPR condition, i.e.

$$\mathrm{Re}[H(\mathrm{e}^{\mathrm{j}\omega})] > 0 \quad \forall \omega \in (-\pi, \pi] \tag{48}$$

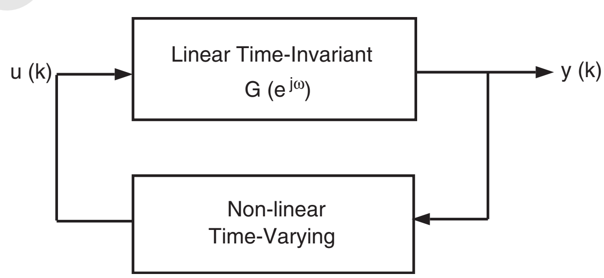

Figure 4. Non-linear time-varying feedback system.

Equation (48) is a sufficient condition. If this condition is not satisfied, the simplified recursive algorithm is not necessarily locally unstable. Several of the most common point spread functions encountered in practice are motion blur, uniform out-of-focus blur, atmospheric turbulence (Gaussian) blur and scatter blur. Motion and out-of-focus blurs do not satisfy the SPR condition. Gaussian and scatter blurs may or may not satisfy the SPR condition depending on their parameters. If a PSF does not satisfy the SPR condition, one needs to implement the recursive algorithm without ignoring the regressor filtering to guarantee stability.

7. SIMULATION RESULTS

The theory developed is supported with two computer experiments in this section. In the first experiment, the simplified recursive algorithm is shown to work for an SPR blur. It is also shown to work for a non-SPR blur in the second experiment. The classical 8-bit grey-scale *Pepper* and *Lena* images were used as true (test) images in the experiments. Histogram equalization was performed on the test images that results in approximately uniformly distributed images. Then, their means were subtracted from the histogram equalized images yielding zero-mean uniformly distributed images. Finally, uniform quantizations having different step-sizes were applied to the zero-mean uniformly distributed images to obtain test images. The performance was tested at 70 dB blurred signal-to-noise ratio (BSNR) defined as

$$\mathrm{BSNR} = 10\log_{10}\left\{\frac{(1/MN)\sum_{m=1}^{M}\ \sum_{n=1}^{N}\ z^2(m,n)}{\sigma_v^2}\right\} \tag{49}$$

where $z(m,n)$ is the noise free blurred image, i.e. $z(m,n) = g(m,n) - v(m,n)$ in (1) and σ_v^2 is the additive noise variance. The improvement in signal-to-noise ratio (ISNR) metric was used for the purpose of testing the performance of the method. This metric is given by

$$\mathrm{ISNR} = 10\log_{10}\left\{\frac{\sum_{m=1}^{M}\ \sum_{n=1}^{N}[f(m,n) - g(m,n)]^2}{\sum_{m=1}^{M}\ \sum_{n=1}^{N}[f(m,n) - \hat{f}(m,n)]^2}\right\} \tag{50}$$

where $f(m,n)$ and $g(m,n)$ are the original and degraded images and $\hat{f}(m,n)$ is the estimated true image. BSNR is at most 50 dB when images are digitally recorded. If BSNR is above 40 dB, the noise is not visible. However, as BSNR goes below 20 dB, the noise becomes more prominent than the blurring and blind image deconvolution methods become useless.

Because the CM cost is non-convex, the method may converge to a local minimum instead of the global minimum of J_{CM} depending on how it is initialized. If there is no *a priori* information about the PSF, the adaptive AR filter is initialized using a zero value for all coefficients (as opposed to a 2-D impulse function initialization in the FIR case). If there is *a priori* information about the PSF, this information may provide a better initialization. Since it was assumed that the PSF is unknown, a 2-D filter with zero coefficients was used as the initial adaptive filter.

Experiment 1

Figure 5 depicts the real part of the 128-point discrete fourier transform (DFT) of the PSF used in this experiment. It is clear that the SPR condition is satisfied by this PSF. Figures 6–9 illustrate the 2, 4, 8, 16-level true (left column), degraded (middle column) and estimated true images (right column) at 70 dB BSNR, respectively. Table I provides the true PSF $h(m,n)$

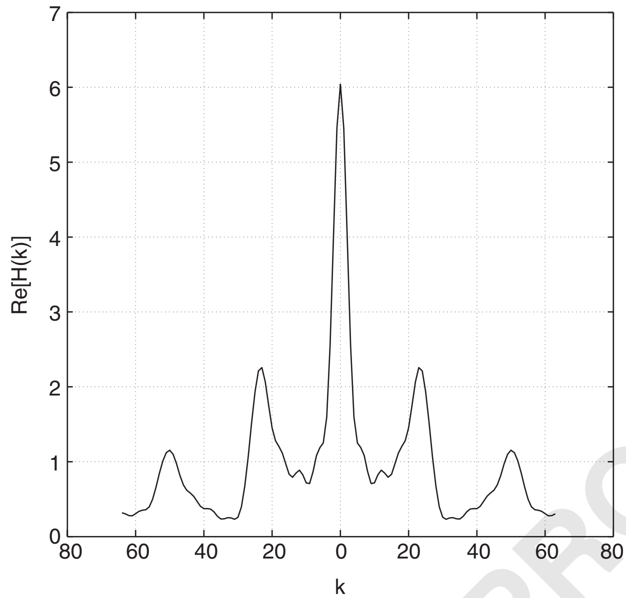

Figure 5. A PSF which satisfies the SPR condition.

Figure 6. Deconvolution result for the SPR PSF, $L = 2$. ISNR = 64.46 dB.

Figure 7. Deconvolution result for the SPR PSF, $L = 4$. ISNR = 45.30 dB.

and coefficients of the adaptive filter at convergence for each level. Adaptive filter coefficients converge to the true PSF well, though the performance worsens as the number of grey level increases. This is an expected result because as the number of grey level (kurtosis) increases, the CM cost surface flattens making convergence of the filter coefficients slow [12].

Figure 8. Deconvolution result for the SPR PSF, $L = 8$. ISNR = 43.54 dB.

Figure 9. Deconvolution result for the SPR PSF, $L = 16$. ISNR = 38.80 dB.

Table I. An SPR PSF and adaptive filter coefficients at convergence for $L = 2, 4, 8, 16$.

		$w(m,n)$			
(m,n)	$h(m,n)$	$L=2$	$L=4$	$L=8$	$L=16$
(0, 0)	1	×	×	×	×
(0, 1)	0.7155	0.7147	0.7003	0.7215	0.7278
(0, 2)	0.3536	0.3533	0.3356	0.3479	0.3611
(0, 3)	0.1707	0.1715	0.1721	0.1583	0.1733
(0, 4)	0.0894	0.0905	0.0777	0.0755	0.0554
(1, 0)	0.7155	0.7148	0.7109	0.7247	0.7239
(1, 1)	0.5443	0.5440	0.5210	0.5501	0.5469
(1, 2)	0.2963	0.2979	0.2814	0.3086	0.3208
(1, 3)	0.1527	0.1542	0.1499	0.1646	0.1834
(1, 4)	0.0831	0.0827	0.0863	0.0887	0.0864
(2, 0)	0.3536	0.3538	0.3649	0.3792	0.3916
(2, 1)	0.2963	0.2974	0.2858	0.3093	0.3251
(2, 2)	0.1925	0.1930	0.1865	0.2004	0.2096
(2, 3)	0.1141	0.1127	0.1088	0.1256	0.1428
(2, 4)	0.0680	0.0671	0.0895	0.0852	0.0754
(3, 0)	0.1707	0.1701	0.1627	0.1805	0.1736
(3, 1)	0.1527	0.1512	0.1457	0.1611	0.1733
(3, 2)	0.1141	0.1121	0.1097	0.1185	0.1117
(3, 3)	0.0775	0.0765	0.0793	0.0802	0.0699
(3, 4)	0.0512	0.0533	0.0707	0.0412	0.0151
(4, 0)	0.0894	0.0910	0.1135	0.0362	0.0287
(4, 1)	0.0831	0.0839	0.1032	0.0673	0.0716
(4, 2)	0.0680	0.0683	0.0846	0.0682	0.0351
(4, 3)	0.0512	0.0517	0.0610	0.0402	0.0018
(4, 4)	0.0370	0.0365	0.0518	−0.0052	−0.0537

Two observations are in order for this experiment. First, the PSF used in this experiment is a 5×5 scatter blur with parameter $\beta = 1$ whose coefficients are given by

$$h(m,n) = \frac{C}{(\beta^2 + (m^2 + n^2))^{3/2}} \tag{51}$$

Blind deconvolution results using the optimum support for the adaptive FIR filter were given in References [9, 13] for this PSF, where the ISNR was in the 20–7 dB range (depending on the number of grey levels in the true image). ISNR is in the 64–38 dB range when using an adaptive AR deconvolution filter (depending on the number of grey levels). The main reason for the improvement is that the adaptive AR filter does not suffer error from a non-optimal support as does the adaptive FIR filter.

Second, it was observed from simulations that convergence of the adaptive AR filter occurs faster than that of the adaptive FIR filter. Even though around 1000 iterations were required for convergence of the adaptive FIR filter, convergence took place after 200 iterations in the adaptive AR filter case. This difference between the convergence speeds of the two cases may be explained by noting that the AR convolution requires fewer coefficients to be updated in each iteration. The optimum support for the adaptive FIR filter was 7×7 (see Reference [13]), so there are 49 adaptive coefficients requiring update at each iteration. However, the optimum support for the adaptive AR filter is 5×5 since the blur is 5×5, so there are only 25 adaptive parameters.

Experiment 2

Figure 10 depicts the real part of the 128-point DFT of the PSF used in this experiment. This time the SPR condition is not satisfied. Figures 11–14 illustrate the 2, 4, 8, 16-level true (left column), degraded (middle column) and estimated true images (right column) at 70 dB BSNR,

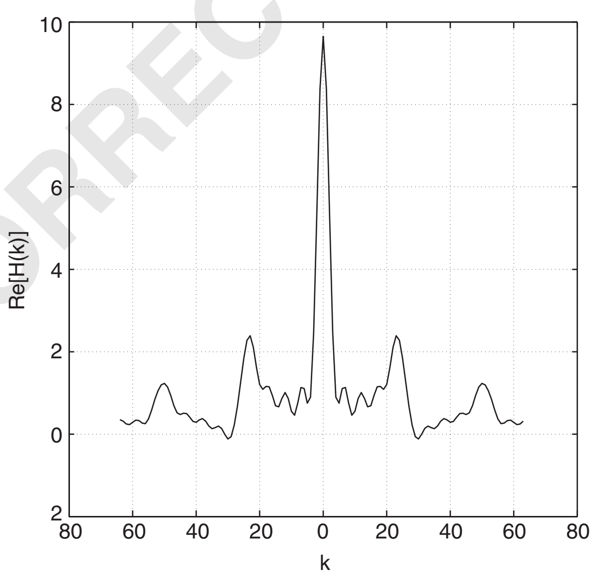

Figure 10. A PSF which violates the SPR condition.

Figure 11. Deconvolution result for the non-SPR PSF, $L = 2$. ISNR = 40.51 dB.

Figure 12. Deconvolution result for the non-SPR PSF, $L = 4$. ISNR = 39.72 dB.

Figure 13. Deconvolution result for the non-SPR PSF, $L = 8$. ISNR = 36.45 dB.

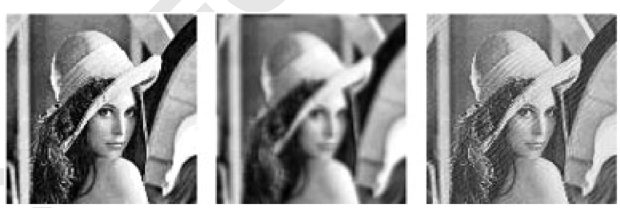

Figure 14. Deconvolution result for the non-SPR PSF, $L = 16$. ISNR = 34.03 dB.

respectively. Table II provides the true PSF $h(m, n)$ and the adaptive filter coefficients at convergence for each level. Adaptive filter coefficients converge to the true PSF reasonably well. The errors between the converged adaptive filters and the PSF are worse than when the SPR conditions was satisfied.

As stated before, the SPR condition is only a 'sufficient' condition for local stability of the simplified recursive algorithm. If the true image has little spectral content at the frequencies where the blur violates the SPR condition, the simplified algorithm is expected to work since the

Table II. An non-SPR PSF and adaptive filter coefficients at convergence for $L = 2, 4, 8, 16$.

		$w(m,n)$			
(m,n)	$h(m,n)$	$L=2$	$L=4$	$L=8$	$L=16$
(0, 0)	1	×	×	×	×
(0, 1)	0.8538	0.8811	0.8780	0.8826	0.8601
(0, 2)	0.5760	0.6147	0.6107	0.6360	0.5673
(0, 3)	0.3536	0.3796	0.3751	0.4249	0.3828
(0, 4)	0.2160	0.2142	0.2072	0.3206	0.3664
(1, 0)	0.8538	0.8711	0.9154	0.9487	0.9200
(1, 1)	0.7401	0.7683	0.7273	0.7863	0.7644
(1, 2)	0.5154	0.5150	0.4641	0.5266	0.5421
(1, 3)	0.3260	0.2921	0.2645	0.3006	0.3992
(1, 4)	0.2037	0.1555	0.1370	0.2439	0.3756
(2, 0)	0.5760	0.5969	0.6290	0.6618	0.6058
(2, 1)	0.5154	0.5010	0.4500	0.4888	0.5379
(2, 2)	0.3852	0.3247	0.3153	0.3162	0.4158
(2, 3)	0.2617	0.1948	0.1902	0.1689	0.2579
(2, 4)	0.1729	0.1387	0.1212	0.1701	0.1860
(3, 0)	0.3536	0.3692	0.3948	0.3867	0.3711
(3, 1)	0.3260	0.2891	0.2824	0.2444	0.3362
(3, 2)	0.2617	0.2149	0.2410	0.1769	0.2426
(3, 3)	0.1925	0.1755	0.1656	0.1179	0.0941
(3, 4)	0.1362	0.1550	0.1425	0.1845	0.0997
(4, 0)	0.2160	0.2577	0.2967	0.2036	0.1959
(4, 1)	0.2037	0.2321	0.2553	0.1001	0.0907
(4, 2)	0.1729	0.2263	0.2400	0.1136	0.0379
(4, 3)	0.1362	0.1978	0.1889	0.1350	0.0230
(4, 4)	0.1028	0.1438	0.1646	0.2254	0.1799

algorithm will go in the right direction most of the time. On the other hand, when the true image has large spectral content at the frequencies where the blur violates the SPR condition, the algorithm will go in the wrong direction most of the time, and eventually will become unstable.

8. CONCLUSIONS

FIR filters are more common than IIR filters because adaptive FIR filters can always be made stable by adjusting the step-size, while adaptive IIR filters may become unstable no matter how small the step-size. Moreover, adaptive algorithms based on FIR filters are usually mathematically more tractable than those based on IIR filters. Nonetheless, there are substantial gains to be made by exploiting the more general IIR structure.

The contribution of this paper is twofold. First, it introduces the use of the CM cost for estimating a grey-scale true image distorted by a LSI blur where the true image and blur are unknown, by minimizing the CM cost using an adaptive 2-D AR filter. The method imposes only mild constraints on the unknown blur and is useful as long as the true image is sub-

Gaussian (the true image dispersion constant is less than 3) and the BSNR is above about 30 dB. Second, we have shown how an adaptive 2-D AR filter may be more suitable than an adaptive 2-D FIR filter for blind image deconvolution in terms of ISNR and convergence speed, at least when both filters are updated by minimizing the CM cost.

One limitation of the AR case is the presence of regressor filtering, which makes realization of the AR method computationally costly. A simplified algorithm that bypasses the regressor filtering was proposed. The SPR condition for the lexicographically ordered PSF was shown to be sufficient condition for local stability when applied to binary images. Unfortunately, some PSFs do not satisfy the SPR condition. Fortunately, this does not necessarily imply instability of the simplified method, which was shown to work for some non-SPR blurs. If the true image has little spectral content at the frequencies where the PSF violates the SPR condition, it is conjectured that the simplified method will remain viable. Otherwise, the complete AR algorithm may be used where the simplified method fails.

This work can be extended in several ways to improve computational aspects of the proposed algorithm and to make it a reliable, practical method for blind image deconvolution. Important areas for further investigation are: (i) overcoming limitations, thus increasing performance of the method, (ii) generalizing local stability analysis to more general situations.

The limitations of the dispersion minimization algorithm are of two kinds: convergence of the adaptive filter to a (bad) local minimum of the CM cost instead of the global minimum, and decreased performance as the true image normalized kurtosis increases. The former is due to the non-convex structure of the CM cost and lack of smart adaptive filter initialization methods. The latter occurs when the kurtosis of the true image increases, for instance when the number of levels increases.

In our experiments, the adaptive AR filter was initialized at zero, which may cause the algorithm to suffer from convergence to a local minimum. Is there an initialization method that guarantees convergence to the global minimum of the CM cost? If so, how can this initialization be chosen? If there is *a priori* information about the blur (for instance, if it is known to be Gaussian) then better initialization strategies are possible. These might keep the adaptive filter from converging to a local minimum of the CM cost, as well as help speed convergence.

Three methods might help increase the performance of the method. First, pixel values of images, in general, do not satisfy the CM assumption. Therefore, better results may be obtained if the more general 'multimodulus cost' [22] is used. Second, if the real-valued true image is represented as a complex-valued image, then an increase in the true image kurtosis implies a smaller deviation from the CM assumption in the complex-valued image. This method also requires a complex-valued adaptive filter that can be implemented using four real-valued adaptive filters, with computational complexity four times that of the real case. Third, suppose that the true image is 8-bit, so its kurtosis is far from the constant modulus assumption. Obtaining a binary (1-bit) image by quantizing the degraded image, and then applying the dispersion minimization method to the binary image would produce better results than applying the method to the degraded image. Next, the adaptive filter at convergence for the binary case could be used to initialize the adaptive filter in the dispersion minimization algorithm on a 2-bit image obtained by quantizing the degraded image, and so on. This 'bootstrapping' initialization scheme might provide much faster convergence and increased performance since at each level a better initialization is used for the adaptive filter compared to the blind zero filter initialization.

APPENDIX A: PROOF OF THEOREM 2

The proof will be given using the following steps.

1. Determine the sate equations that describe the error system $[\mathbf{X}_{1,j}(k), \mathbf{X}_{2,j}]^{\mathrm{T}}$.
2. Linearize the error system about the solution $W(z) = H(z) - 1$, which is equivalent to $\mathbf{X}_{1,j}(k) = \mathbf{X}_{2,j} = \mathbf{0}$.
3. Apply the 'Hyperstability theorem' to show that if the PSF is SPR, then the linearized error system is exponentially stable. Under this condition, the simplified recursive algorithm converges to $W(z) = H(z) - 1$.

Step 1: For a binary image ($\gamma = 1$), the recursive algorithm using simplified updates is given by

$$w_{j+1}(l) = w_j(l) + \mu \hat{f}_j(k)\hat{f}_j(k-1)(\hat{f}_j^2(k) - 1), \quad 1 \leqslant l \leqslant AB - 1 \tag{A1}$$

Note that since $f(k) = \pm 1$, $\hat{f}_j^2(k) - 1$ can be written as

$$\hat{f}_j^2(k) - 1 = (\hat{f}_j(k) - 1)(\hat{f}_j(k) + 1) = (\hat{f}_j(k) - f(k))(\hat{f}_j(k) + f(k)) \tag{A2}$$

Let $e_j(k)$ and $\tau_j(k)$ be the estimation error and the time varying factor for the kth pixel:

$$e_j(k) := f(k) - \hat{f}_j(k) \tag{A3}$$

$$\tau_j(k) := \tfrac{1}{2}(\hat{f}_j^2(k) + \hat{f}_j(k)f(k)) \tag{A4}$$

Then, the simplified recursive algorithm given in Equation (A1) can be expressed as

$$w_{j+1}(l) = w_j(l) - \mu \hat{f}_j(k-l)e_j(k)\tau_j(k) \tag{A5}$$

State variable equations for $\mathbf{X}_{1,j}(k)$ that describes the dynamics of the estimation errors will be derived first. Recall that the degraded image $g(k)$ is given by

$$g(k) = \sum_{i=0}^{AB-1} h(i)f(k-i), \quad h(0) = 1 \tag{A6}$$

Therefore, the true image $f(k)$ can be expressed in terms of the degraded image $g(k)$ as

$$f(k) = g(k) - \sum_{i=1}^{AB-1} h(i)f(k-i) \tag{A7}$$

From (A7) and (24), the estimation error $f(k) - \hat{f}_j(k)$ can be written as

$$f(k) - \hat{f}_j(k) = -\sum_{i=1}^{AB-1} h(i)f(k-i) + \sum_{i=1}^{AB-1} w_j(i)\hat{f}_j(k-i) \tag{A8}$$

Since adding and subtracting $\sum_{i=1}^{AB-1} h(i)\hat{f}_j(k-i)$ to the right-hand side of (A8) does not change its value, (A8) can also be written as

$$f(k) - \hat{f}_j(k) = -\sum_{i=1}^{AB-1} h(i)[f(k-i) - \hat{f}_j(k-i)] - \sum_{i=1}^{AB-1} [h(i) - w_j(i)]\hat{f}_j(k-i) \tag{A9}$$

From Equation (38), $\mathbf{X}_{1,j+1}(k+1)$ is given by

$$\mathbf{X}_{1,j+1}(k+1) = [f(k) - \hat{f}_j(k), \ldots, f(k-AB+2) - \hat{f}_j(k-AB+2)]^{\mathrm{T}} \tag{A10}$$

Define the following matrix and vectors:

$$H := \begin{bmatrix} -h(1) & -h(2) & \dots & & -h(AB-1) \\ 1 & 0 & \dots & & 0 \\ 0 & & & & \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix} \tag{A11}$$

$$\hat{\mathbf{f}}_j(k) := [\hat{f}_j(k-1), \hat{f}_j(k-2), \dots, \hat{f}_j(k-AB+1)]^{\mathrm{T}} \tag{A12}$$

$$\mathbf{h} := [-h(1), -h(2), \dots, -h(AB-1)]^{\mathrm{T}} \tag{A13}$$

$$\mathbf{b} := [1, 0, \dots, 0]^{\mathrm{T}} \tag{A14}$$

Then, $\mathbf{X}_{1,j+1}(k+1)$ is given by

$$\mathbf{X}_{1,j+1}(k+1) = H\mathbf{X}_{1,j}(k) + \mathbf{b}\hat{\mathbf{f}}_j^{\mathrm{T}}(k)\mathbf{X}_{2,j} \tag{A15}$$

which is the state equation for $\mathbf{X}_{1,j}(k)$, where $\mathbf{X}_{2,j}$ is given by (39). Now, the state equation for $\mathbf{X}_{2,j}$ that describes the dynamics of the coefficient errors will be derived. Observe that by (A8), $e_j(k)$ is equal to

$$e_j(k) = \mathbf{h}^{\mathrm{T}}\mathbf{X}_{1,j}(k) - \hat{\mathbf{f}}_j^{\mathrm{T}}(k)\mathbf{X}_{2,j} \tag{A16}$$

Therefore, Equation (A5) can be written as

$$w_{j+1}(l) = w_j(l) - \mu\hat{f}_j(k-l)\tau_j(k)(\mathbf{h}^{\mathrm{T}}\mathbf{X}_{1,j}(k) - \hat{\mathbf{f}}_j^{\mathrm{T}}(k)\mathbf{X}_{2,j}) \tag{A17}$$

Subtracting both sides of (A17) from $h(l)$, writing the resulting expression for $1 \leqslant l \leqslant AB-1$, and using the definition of $\mathbf{X}_{2,j}$ gives

$$\mathbf{X}_{2,j+1} = \mu\tau_j(k)\hat{\mathbf{f}}_j(k)\mathbf{h}^{\mathrm{T}}\mathbf{X}_{1,j}(k) + (I - \mu\tau_j(k)\hat{\mathbf{f}}_j(k)\hat{\mathbf{f}}_j^{\mathrm{T}}(k)) \tag{A18}$$

In summary, the error system describing the dynamics of the recursive algorithm is given by (A15) and (A18) which are

$$\mathbf{X}_{1,j+1}(k+1) = H\mathbf{X}_{1,j}(k) + \mathbf{b}\hat{\mathbf{f}}_j^{\mathrm{T}}(k)\mathbf{X}_{2,j} \tag{A19}$$

$$\mathbf{X}_{2,j+1} = \mu\tau_j(k)\hat{\mathbf{f}}_j(k)\mathbf{h}^{\mathrm{T}}\mathbf{X}_{1,j}(k) + (I - \mu\tau_j(k)\hat{\mathbf{f}}_j(k)\hat{\mathbf{f}}_j^{\mathrm{T}}(k))\mathbf{X}_{2,j} \tag{A20}$$

Step 2: Local stability of maps (A19) and (A20) about the solution $\mathbf{X}_{1,j}(k) = \mathbf{X}_{j,2} = \mathbf{0}$ is determined by linearizing the maps about this solution. The linearized maps are given by

$$\mathbf{X}_{1,j+1}(k+1) = H\mathbf{X}_{1,j}(k) + \mathbf{b}\mathbf{f}^{\mathrm{T}}(k)\mathbf{X}_{2,j} \tag{A21}$$

$$\mathbf{X}_{2,j+1} = \mu\mathbf{f}(k)\mathbf{h}^{\mathrm{T}}\mathbf{X}_{1,j}(k) + (I - \mu\mathbf{f}(k)\mathbf{f}^{\mathrm{T}}(k))\mathbf{X}_{2,j} \tag{A22}$$

which can be compactly written as

$$\begin{bmatrix} \mathbf{X}_{1,j+1}(k+1) \\ \mathbf{X}_{2,j+1} \end{bmatrix} = \begin{bmatrix} H & \mathbf{b}\mathbf{f}^{\mathrm{T}}(k) \\ \mu\mathbf{f}(k)\mathbf{h}^{\mathrm{T}} & (I - \mu\mathbf{f}(k)\mathbf{f}^{\mathrm{T}}(k)) \end{bmatrix} \begin{bmatrix} \mathbf{X}_{1,j}(k) \\ \mathbf{X}_{2,j} \end{bmatrix} \tag{A23}$$

where $\mathbf{f}(k) := [f(k-1), f(k-2), \ldots, f(k-AB+1)]^{\mathrm{T}}$.

Step 3: Define the following signal:

$$u_j(k) := \sum_{i=1}^{AB-1} [h(i) - w_j(i)]\, \hat{f}_j\,(k-i) \tag{A24}$$

Given (39) and (A12), $u_j(k)$ is equal to

$$u_j(k) = \mathbf{X}_{2,j}^{\mathrm{T}} \hat{\mathbf{f}}_j(k) \tag{A25}$$

Consider the closed-loop system depicted in Figure A1 with input $u_j(k)$ and output $e_j(k)$. Exponential stability of $u_j(k)$ and $e_j(k)$ to the origin is equivalent to exponential stability of (A23) to zero since $e_j(k) = u_j(k) = 0$ gives $\hat{f}_j(k) = f(k)$, $w_j(k) = h(k)$, which is the desired result. From the 'Hyperstability theorem' (Theorem 1), $e_j(k)$ and $u_j(k)$ are exponentially stable to the origin if the transfer function from $u_j(k)$ to $e_j(k)$ is SPR. Given (A8), (A13) and (A24), $e_j(k)$ is equal to

$$e_j(k) = \mathbf{h}^{\mathrm{T}} \mathbf{X}_{1,j}(k) + u_j(k) \tag{A26}$$

Given (A25), (A21) can be written as

$$\mathbf{X}_{1,j+1}(k+1) = H\mathbf{X}_{1,j}(k) + \mathbf{b}u_j(k) \tag{A27}$$

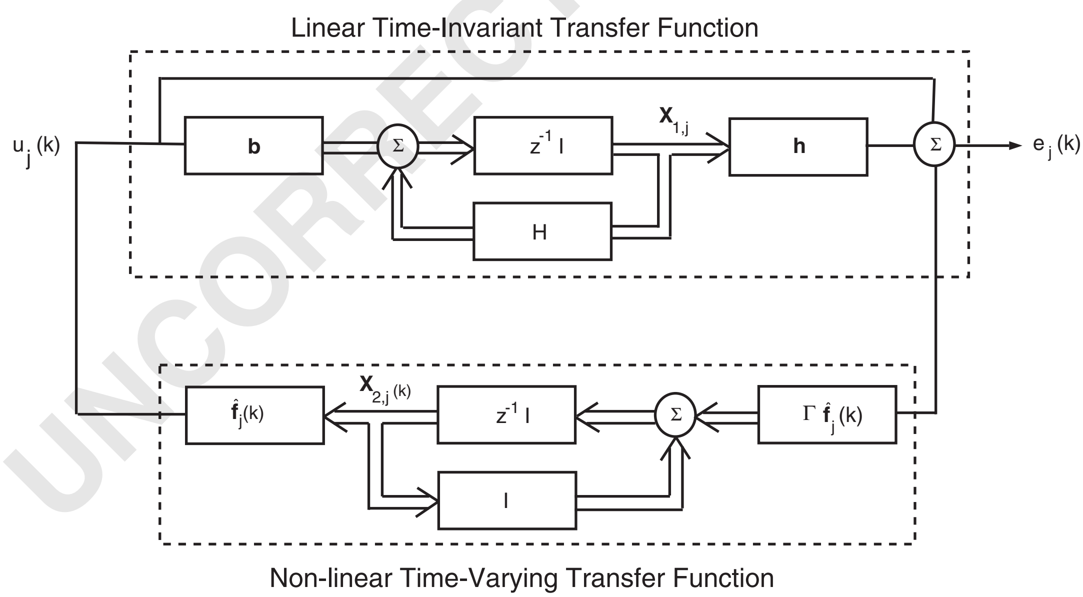

Figure A1. The recursive image deconvolution parameter and estimation error closed-loop system.

From (A26) and (A27), the transfer function from $u_j(k)$ to $e_j(k)$ is equal to

$$\mathbf{h}^{\mathrm{T}}(zI - H)^{-1}\mathbf{b} + 1 = \frac{1}{1 + \sum_{i=1}^{AB-1} h(i)z^{-i}} = \frac{1}{H(z)} \tag{A28}$$

Consequently, the linearized system (A23) is exponentially stable to the origin (equivalently, the simplified recursive algorithm (34) with $\varphi_j(k)$ given as in (37) is locally stable to $W(z) = H(z) - 1$) if

$$\mathrm{Re}\left[\frac{1}{H(\mathrm{e}^{\mathrm{j}\omega})}\right] > 0 \quad \forall \omega \in (-\pi, \pi] \tag{A29}$$

or by Lemma 1

$$\mathrm{Re}[H(\mathrm{e}^{\mathrm{j}\omega})] > 0 \quad \forall \omega \in (-\pi, \pi] \tag{A30}$$

provided the true image $f(k)$ is persistently exciting.

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