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Sensor Placement for On-Orbit Modal Identification via a Genetic Algorithm

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A variant of the genetic algorithm is used to place sensors optimally on a large space structure for the purpose of modal identification. The selection and reproduction schemes of the genetic algorithm are modified, and a new operator called forced mutation is introduced. These changes are shown to improve the convergence of the algorithm and to lead to near-optimal sensor locations. Two practical examples are investigated: sensor placement for an early version of the space station and an individual space station photovoltaic array. Simulated results are also compared with previous results obtained by the effective independence method. The genetic algorithm-based sensor configuration estimates the target mode response more accurately.

Introduction

ON-ORBIT system identification is a vital component of the successful operation of large space structures (LSS). The number of sensors used on the LSS for system identification is limited due to weight and cost considerations. The sensors must be placed in an optimal fashion so that the modal characteristics of the LSS can be identified as accurately as possible. To determine the optimal sensor locations, extensive prelaunch analysis must be performed, since the sensors cannot be easily moved on orbit.

A vast amount of literature has been produced dealing with sensor placement for system identification and control. Much of the work deals with distributed parameter systems.[1-5] A relatively small number of papers[6,7] have considered sensor placement for structural parametric identification. Other papers, such as Ref. 8, have investigated sensor placement for verification of large dynamical systems. However, only Ref. 9 has specifically considered the sensor placement problem from the standpoint of a structural dynamicist who must use the data collected from the sensors to validate an LSS finite element model (FEM) using test-analysis correlation techniques.[10]

In Ref. 9, an efficient method called effective independence (EfI) is presented to place a small number of sensors for modal identification of an LSS. Based on the prelaunch FEM, a set of target modes is selected for identification. The target mode partitions must be linearly independent or correlation methods, which compare test and analysis mode shapes, will not be able to spatially differentiate between the modes. The EfI method casts the linear independence problem in the form of a target mode response estimation problem. The sensor locations that produce the best target mode response estimate also produce linearly independent target mode shape partitions. An initial candidate set of sensor locations is selected. The approach then ranks the candidate locations based on their contribution to the linear independence of the corresponding FEM target mode partitions. Locations that do not contribute are removed from the candidate set. In an iterative manner, the initial candidate set of sensor locations is reduced to the number of available sensors. The EfI method is efficient yet suboptimal. The errors between the real target mode response and the estimated target mode response obtained by sensors placed at the locations determined by the EfI method cannot be guaranteed to be minimum.

The genetic algorithm (GA)[11,12] has been successfully applied to a variety of system identification problems. For example, the GA has been applied to the delay estimation of sampled signals[13,14] and to the parameter estimation of linear adaptive filters.[15] It has also been applied to the training of feedforward neural networks.[16,17] The impact of tuning factors on the performance of the GA has been studied extensively in Ref. 18. In this paper, the GA will be applied to the same sensor placement problem presented in Ref. 9. Since the GA is a globally optimal method, the sensor locations estimated by the GA can find the global minimum of the error between the real target mode response and the estimated response.

To apply the GA to the problem of sensor placement for modal identification of an LSS, each of the possible sensor locations is represented as an integer (the "gene"), and k such sensor locations are concatenated into an integer string (the "chromosome"). Each chromosome represents a possible set of sensor locations. The fitness of each chromosome is defined as the determinant of the corresponding Fisher information matrix (FIM).[19] Thus, the quality of any given set of sensor locations is readily discernible. The goal of the GA is to find the "best" chromosome, that is, the set of sensor locations that maximizes the determinant of the FIM.

It will be shown in this paper that the parent selection and reproduction strategies in Refs. 11 and 12 hinder the GA from finding the optimal sensor locations for modal identification of the LSS. Both strategies will be modified so that the GA can lead to an effective and useful solution. A new operator called forced mutation, which is performed on any chromosome with redundant sensor locations, is introduced. Mathematically, it is shown that forced mutation improves the convergence of the algorithm. Two practical examples are tested. First, the GA is applied to select the best sensor locations for modal identification of an early version of the space station. Second, it is applied to select the best sensor locations on an individual photovoltaic (PV) array. Numerically, it will be shown that the determinant of the corresponding FIM based on the sensor locations estimated by the GA is larger than the one based on the sensor locations estimated by the EfI method. The GA thus provides a superior set of sensor locations at the expense of additional prelaunch computational complexity.

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Problem Statement

Because of weight and cost considerations, only a small number of sensors can be placed on the LSS for modal identification. The set of target modes is assumed to include all modes that are strongly excited by the actuator configuration. The set of candidate locations for sensor placement is chosen large enough such that all of the important dynamics within the target modes are included. Assume that there are m candidate locations on the LSS and n target modes to be identified. Let $y \in R^{m\times 1}$ be the vector containing sensor outputs at all of the possible candidate locations. Let $\Phi \in R^{m\times n}$ be the matrix of FEM target modes partitioned to the candidate sensor locations and let $q \in R^{n\times 1}$ be the vector of target modal coordinates. Then

$$y = \Phi q + w \tag{1}$$

where w denotes the vector of measurement noise that is assumed to be zero-mean Gaussian noise.

The set of candidate locations is chosen so that the column vectors of Φ are linearly independent. This spatial independence must be maintained by the final selected sensor configuration. Linear independence of the target mode partitions implies that at any time t the outputs of the sensors can be sampled and the target mode dynamic response can be estimated. Theoretically, only n sensors are needed to identify n target modes. Practically, the number of sensors placed on the LSS must be more than n to account for engineering uncertainty.

Suppose k sensors are to be placed on the LSS at k distinct locations. Note that placing the sensors at different locations will maximize the linear independence of the target modes for the fewest number of locations. Then, for the ith set of the selected k locations, the outputs at the corresponding sensors are described by

$$y_i = \phi_i q + \omega_i \tag{2}$$

where $y_i \in R^{k\times 1}$ is the vector of outputs at these selected sensors, $\phi_i \in R^{k\times n}$ is the matrix containing k rows of Φ corresponding to the selected k sensor locations, and $\omega_i \in R^{k\times 1}$ denotes the measurement noise at these sensors. Note that ω_i is still zero mean and Gaussian. The least-squares estimate of the target modal coordinates for the ith set of selected sensor locations is

$$\hat{q}_i = (\phi_i^T\phi_i)^{-1}\phi_i^T y_i \tag{3}$$

The sensor placement problem thus consists of choosing the best set (say, set j) of sensor locations from among the $\binom{m}{k}$ possible placements so that

$$\|\hat{q}_j - q\|^2 \le \|\hat{q}_i - q\|^2, \qquad \forall 1 \le i \le \binom{m}{k} \tag{4}$$

Referring to Eq. (2), since ω_i is Gaussian and zero-mean, the probability density function is given by

$$f(y_i) = (2\pi)^{-k/2}[\det(F_{y_i})]^{-1/2}\exp[(-1/2)(y_i - m_{y_i})^T F_{y_i}^{-1}(y_i - m_{y_i})] \tag{5}$$

where

$$m_{y_i} = E(y_i) = \phi_i q \tag{6}$$

and

$$\begin{aligned} F_{y_i} &= E[(y_i - m_{y_i})(y_i - m_{y_i})^T] \\ &= E(\omega_i\omega_i^T) \end{aligned} \tag{7}$$

Referring to Eq. (3), it is known that the probability density function of $\hat{q}_i$ is also Gaussian, i.e.,

$$f(\hat{q}_i) = (2\pi)^{-k/2}[\det(F_{\hat{q}_i})]^{-1/2}\exp[-1/2(\hat{q}_i - m_{\hat{q}_i})^T F_{\hat{q}_i}^{-1} \times (\hat{q}_i - m_{\hat{q}_i})] \tag{8}$$

where

$$\begin{aligned} m_{\hat{q}_i} &= (\phi_i^T\phi_i)^{-1}\phi_i^T m_{y_i} \\ &= q \end{aligned} \tag{9}$$

and

$$F_{\hat{q}_i} = (\phi_i^T\phi_i)^{-1}\phi_i^T F_{y_i}\phi_i(\phi_i^T\phi_i)^{-1} \tag{10}$$

With the probability density function as in Eq. (8), it is shown in Ref. 20 that the confidence region of $\hat{q}_i$ can be described by the interior of the hyperellipsoid

$$(\hat{q}_i - q)^T F_{\hat{q}_i}^{-1}(\hat{q}_i - q) = c^2 \tag{11}$$

where c is a constant related to the scale of confidence. The matrix $F_{\hat{q}_i}^{-1}$ is also known as the Fisher information matrix. The volume V_i of the hyperellipsoid in Eq. (11) is given by

$$V_i = \pi^{k/2}c^k[\det(F_{\hat{q}_i})]^{1/2}[\Gamma(k/2+1)]^{-1} \tag{12}$$

where $\Gamma(\cdot)$ denotes the gamma function and $\det(\cdot)$ denotes the determinant. To minimize $\|\hat{q}_i - q\|^2$, the volume of the confidence region in Eq. (12) should be minimized. This is equivalent to minimizing the determinant of $F_{\hat{q}_i}$ since the other terms in Eq. (12) are all constants. As in Ref. 9, the measurement noise in this paper is assumed to be uncorrelated between sensors and is assumed to be of equal variance σ^2 at each sensor. Therefore, referring to Eqs. (7) and (10),

$$\begin{aligned} \det(F_{\hat{q}_i}) &= \det[\sigma^2(\phi_i^T\phi_i)^{-1}] \\ &= \sigma^{2k}[\det(\phi_i^T\phi_i)]^{-1} \end{aligned} \tag{13}$$

Thus, maximizing the determinant of $\phi_i^T\phi_i$ is equivalent to minimizing the determinant of $F_{\hat{q}_i}$. In the sequel, the matrix $\phi_i^T\phi_i$ will be referred to as the FIM. The GA is used in this paper to search for the best set of k sensor locations so that the determinant of the FIM is maximized. Note that a simple search is impractical due to the magnitude of $\binom{m}{k}$.

Although not investigated in this paper, it is a straightforward extension to consider measurement noise that is Gaussian and zero mean but with arbitrary correlation matrix. Instead of applying the GA to search for the best sensor locations corresponding to the maximum determinant of $\phi_i^T\phi_i$ as discussed earlier, the sensor locations corresponding to the minimum determinant of $F_{\hat{q}_i}$ are searched for by the GA.

Genetic Algorithm

In this section, the implementation of the GA for solving the sensor placement problem is described in detail. In the GA, there are three main operators: parent selection, crossover, and mutation. The full scheme, including these three operators and their modifications, is described as follows.

A. Initialization and Fitness Value

Suppose a gene pool of N chromosomes is explored in every generation of the GA. Each chromosome is encoded as a string of integers that correspond to the estimated sensor locations. Since k sensors are to be placed at k different locations among m candidate choices, each chromosome consists of k genes and each gene is an integer ranging from 1 to m. Suppose the jth chromosome is a set of integers $(d_1, \ldots, d_k)$, where $1 \le d_s \le m$. Let ϕ_{ij} be the matrix corresponding to the jth chromosome in the ith generation, then

$$\phi_{ij} = [r_{d_i}^T, \ldots, r_{d_k}^T]^T \tag{14}$$

where r_{d_s} denotes the d_sth row of matrix Φ. The fitness value f_{ij} associated with this chromosome is defined as

$$f_{ij} = \det(\phi_{ij}^T \phi_{ij}) \tag{15}$$

The best sensor locations estimated in generation i are defined by the chromosome corresponding to the largest fitness value

$$f_i = \max_j (f_{ij}) \tag{16}$$

The GA is used to search for the best sensor locations so that f_i converges to the maximum possible value.

Initially, the estimated sensor locations are randomly assigned. Therefore, the GA starts with a collection of chromosomes that are stings of distinct random integers uniformly distributed between 1 and m.

B. Parent Selection

The selection of parent chromosomes for mating is based on the notion of "fitness," which governs the extent to which an individual can influence future generations. The parent selection strategy of the GA in Refs. 11 and 12 is to let the jth chromosome in the ith generation be selected for mating with probability

$$p_{ij} = f_{ij} \Big/ \sum_{s=1}^{N} f_{is} \tag{17}$$

However, there are only a few sets of combinations of rows in Φ that result in large determinants of the FIM. After several generations, the largest fitness value tends to be much larger than that of other chromosomes. According to the parent selection scheme in Eq. (17), the best chromosome will be selected as a parent repeatedly. This causes a rapid decrease in the diversity of the gene pool, that is, the chromosomes in the succeeding generations tend to become homogeneous. Consequently, the only way of improving the fitness value in the next generation is through mutation (random changes on some genes in each chromosome).

In the sensor location problem, the fitness value is extremely sensitive to changes of the genes. For instance, the fitness values of two chromosomes can be significantly different even though they differ in only one gene. Therefore, on one hand, it is difficult to further improve the fitness value because of the homogeneity of the gene pool. On the other hand, the fitness value is usually decreased by mutation, and it often requires several generations to return the fitness value back to its premutation level. Drastic oscillations of the fitness value thus tend to occur once the fitness value is close to the maximum.

Since the improvement of the fitness value requires that the better chromosomes with larger fitness values be allowed to mate, we modify the GA to ensure that these chromosomes are mated in every generation. In each generation, all of the best D chromosomes corresponding to the highest fitness values are allowed to survive into the next generation and serve as potential parents. These D chromosomes are randomly chosen as the parents with equal probability. With this modified parent selection scheme, the problem of consistently choosing parents with larger fitness values than others is avoided. Furthermore, since the best D parents are allowed to live into the next generation, the fitness value of the next generation is greater or equal to the one in the current generation. In other words, the best fitness value in every generation is a monotonically increasing sequence. The oscillation of the fitness value at convergence is avoided, and the real maximum will be eventually attained. In the next section, the effects of the modified parent selection scheme are illustrated through numerical simulation.

C. Mating

Among the D parents, pairs are randomly selected for mating, which is carried out via a crossover procedure that mimics biological mating. Basically, crossover combines the features of two parent chromosomes to form two new children chromosomes. The operation of crossover is briefly illustrated as follows. Let parents 1 and 2 be chromosomes of the form

Parent 1: $x\ x\ x\ x\ x\ x\ x$
Parent 2: $y\ y\ y\ y\ y\ y\ y$

If the randomly assigned splice point is set at the position between the third and fourth genes, then two children would be given by

Child 1: $x\ x\ x\ y\ y\ y\ y$
Child 2: $y\ y\ y\ x\ x\ x\ x$

Even though identical genes are not allowed to coexist when assigning a random number to each gene of the chromosome initially, it is still possible to have pairs of identical genes in the chromosome after crossover. For instance, assume that parent 1 is (1, 10, 15, 20, 30), parent 2 is (1, 2, 3, 10, 15), and the splice point for crossover is still set at the position between gene 3 and 4. One of the children would be (1, 10, 15, 10, 15). Apparently, two pairs of identical genes can occur in the child chromosomes even though no identical genes are in either of the parents. For the chromosomes with r pairs of identical genes, the effective fitness value is calculated as the determinant of $\phi'^T\phi'$ where $\phi' \in R^{(k-r)\times n}$ is the matrix corresponding to $(k-r)$ distinct genes in the chromosome.

D. Mutation

The previous example shows that it is possible to generate chromosomes with identical genes via the crossover procedure. The next theorem shows that the effective fitness value of the chromosome with pairs of identical genes is less than or equal to the fitness value of the chromosome with one of the identical genes replaced by any value different from the other genes.

Theorem 1: For every positive definite $A = \Phi^T\Phi \in R^{n\times n}$, let $r_i \in R^{1\times n}$ be the ith row vector of Φ and $B = A - r_i^T r_i$; then

$$\det(B) = \det(A)(1 - E_{Di}) \tag{18}$$

where $0 \le E_{Di} \le 1$.

Proof: See the Appendix.

Replacing one of the identical genes with any value different from the other genes is analogous to exerting a forced mutation on such genes. From the preceding theorem, the probability of improving fitness value is increased by this forced mutation.

Along with the forced mutation, the idea of natural mutation is also introduced to improve the convergence of the fitness value. Mathematically, it is shown that over a period of

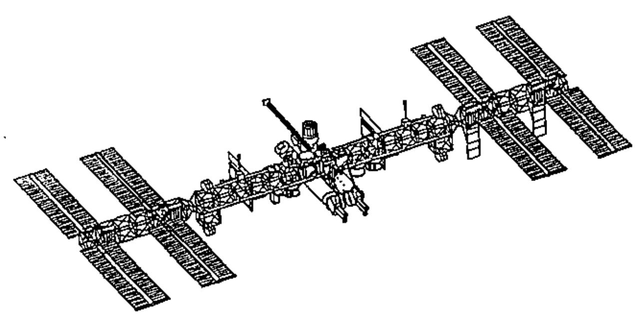

Fig. 1 Early version of proposed space station.

several generations, the chromosomes tend to become more and more homogeneous as one chromosome begins to dominate.[11] A feature of natural mutation is often introduced to guard against premature convergence to a nonoptimal solution. Natural mutation is a procedure of replacing the value of any individual gene with a random number uniformly distributed between 1 and m with probability p_m. Since identical genes are to be avoided, the value assigned to the mutated gene is monitored so that it is different from the rest of the genes in the chromosome.

The algorithm is summarized as follows:

1) Initialize N chromosomes. Set index = 0 and no-improvement tolerance L.

2) Construct ϕ_{ij} as in Eq. (14) for each chromosome and calculate the associated fitness value f_{ij} as in Eq. (15).

3) Select D chromosomes corresponding to the largest fitness value among N chromosomes as the parents. Pass them into the next generation.

4) Mate D parents and generate $N-D$ children. Invoke mutation along with the crossover procedure.

5) The numbers contained in the genes of the best chromosome with largest fitness value are the estimated sensor locations in the current generation. If the highest fitness value in the current generation is the same as the one in the previous generation, index = index + 1; otherwise index = 0.

6) If index $>L$, stop; otherwise go back to step 2 and continue.

Let G be the total number of generations required for the GA to estimate the optimal sensor locations so that the maximum fitness value is achieved. If H_a and H_b are the numbers of combinations of sensor locations processed by the GA and by a random search method, respectively, then $H_a \leq NG$ and $H_b = \binom{n}{k}$. The inequality in H_a occurs because some genes are present in more than one generation, and these need not be re-evaluated each time. It will be shown in the next section that the ratio H_a/H_b is very small. In other words, GA is an efficient optimization method for the sensor placement problem.

Numerical Examples

In this section, the GA is to be applied to select the best sensor locations for modal identification of a space station and a PV array. An early version of the space station will be considered, as shown in Fig. 1, where the main truss and the PV arrays of the space station are illustrated. The optimal solutions obtained by the GA are compared with the suboptimal ones determined by the EfI method in Ref. 9. For the convenience of notation, the genetic algorithm presented in Refs. 11 and 12 will be designated GA1, whereas the algorithm presented in this paper with a modified parent selection and children reproduction scheme will be called GA2. The convergence of the fitness value using GA1 and GA2 is also investigated in the following examples. For both of the examples, the tuning parameters for the GA are set as

N = number of chromosomes in each generation = 100
D = number of parents selected in each generation = 30
p_m = probability of mutation = 0.06
L = no-improvement tolerance = 200

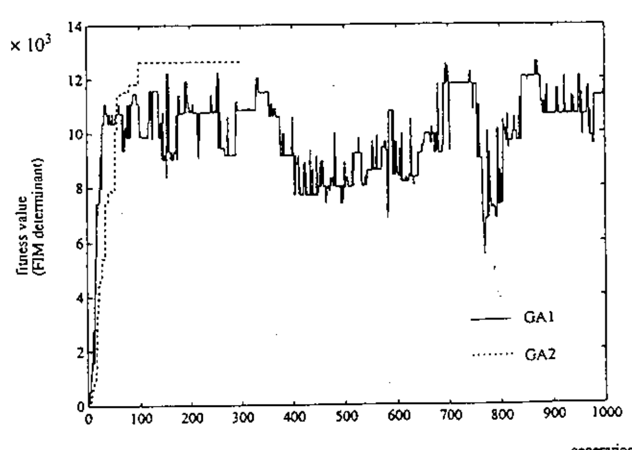

Fig. 2 Convergence of fitness value using GA1 and GA2 algorithms for placement of 10 sensors to identify 7 space station target modes, example 1.

Table 1 Frequencies and descriptions of early version space station target modes, example 1

Mode	Frequency, Hz	Description
21	0.124	First-order bending about x
29	0.144	First-order bending about z
33	0.158	First-order bending about z
34	0.160	First-order bending about x
38	0.331	Second-order bending about x
43	0.488	Second-order bending about z
44	0.513	Second-order bending about x

Table 2 Comparison of simulation results using GA2 and EfI for early version space station, example 1

	10 sensors		20 sensors	
	Largest fitness value	No. of sensor locations different	Largest fitness value	No. of sensor locations different
GA2	1.2652e4	1	8.6236e5	1
EfI	1.2594e4		7.8711e5	

Example 1

In this example, the GA is applied to select the best sensor locations for modal identification of the space station. For demonstration purposes, seven fundamental FEM bending modes are selected for identification. Table 1 lists the frequencies and descriptions of these target modes. The top 200 FEM degrees of freedom based on kinetic energy are selected as the initial candidate set of sensor locations. The kinetic energy of the ith degree of freedom in the jth target mode is given by the relation

$$K_{ij} = \Phi_{ij} \sum_{s=1}^{n_t} M_{is} \Phi_{sj} \tag{19}$$

where Φ_{ij} is the modal coefficient corresponding to the ith degree of freedom in the jth target mode, n_t is the total number of degrees of freedom in the FEM representation, and M is the FEM mass matrix. Since several of the degrees of freedom in the candidate set are located on common rigid bodies within the FEM, the set was eventually reduced to 187 independent candidate locations. It is assumed that this candidate set is large enough to adequately describe the target modes. Suppose 10 sensors are to be placed on the space station to independently identify the target modes. The GA is to be used to search a 187×7 matrix, out of which 10 rows are to be selected so that the determinant of the corresponding FIM is a maximum. The convergence of the fitness value using GA1 and GA2 is represented by solid and dashed lines, respectively, in Fig. 2. It is observed that the convergence of the fitness value using GA1 oscillates in the neighborhood of the real maximum yet never reaches it, whereas the fitness value using GA2 is monotonically increasing and finally reaches the maximum. Clearly, GA2 outperforms GA1. In addition, it takes only 298 generations for GA2 to obtain the maximum. Therefore, $H_a = 29{,}800$ whereas $H_b = \binom{187}{10} = 1.128 \times 10^{16}$. The ratio $H_a/H_b = 2.642 \times 10^{-12}$ indicates that GA2 is a very efficient optimization method for this sensor placement problem. In Fig. 3, the 10 optimal sensor locations selected using GA2 are illustrated.

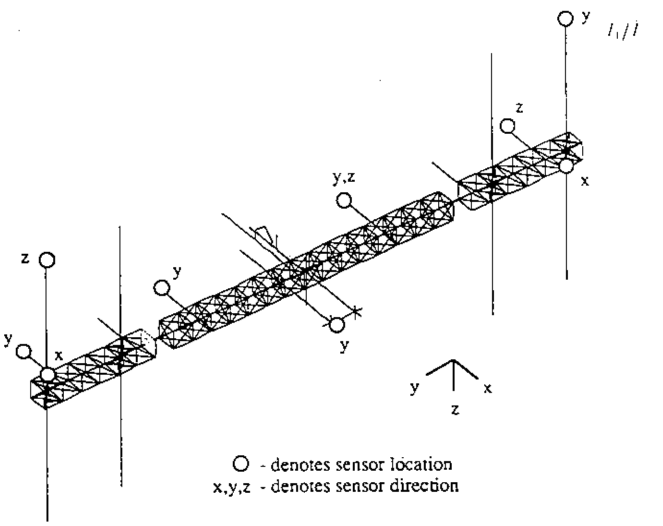

Fig. 3 Ten optimum sensor locations selected by GA2 algorithm for identification of 7 space station target modes, example 1.

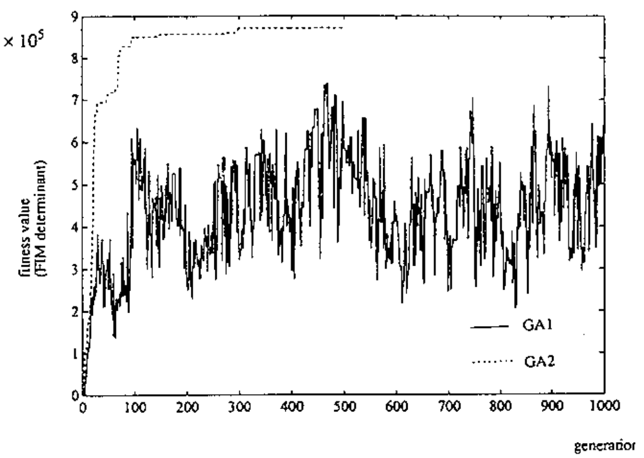

Fig. 4 Convergence of fitness value using GA1 and GA2 algorithms for placement of 15 sensors to identify 7 PV array target modes, example 2.

Finally, GA2 is compared with the EfI method of sensor placement. Using the same example, each of the methods is applied to select the best 20 and then the best 10 sensor locations for identification of the seven target modes. The results of these two methods are compared in Table 2. In both cases, the sensor locations estimated by GA2 correspond to larger fitness values than the sensor locations estimated by the EfI method. The tradeoff is that it takes more computer time for GA2 to obtain the globally optimal sensor locations. To determine 20 and 10 sensor locations, the running time on an IBM 386 PC compatible with coprocessor is 3325 and 1490 s, respectively, for GA2, compared with 97 and 99 s, respectively, for the EfI method.

Example 2

In this example, the GA is further applied to the modal identification of an individual space station PV array. A modal analysis was first performed for a finite element representation of an array resulting in 100 mode shapes with frequencies below 1.69 Hz. These modes were calculated with the PV array fixed at its base, which is consistent with the connection of the array to the space station. Seven modes that possess large loads at the base of the array are selected as target modes. The frequencies and descriptions of these seven target modes are listed in Table 3. The candidate set of sensor locations consists of three mutually perpendicular translations at 107 node points evenly distributed over the array. There are thus 321 candidate sensor locations to be searched. Algorithms GA1 and GA2 are applied to place 15 sensors on the PV array. The convergence of the fitness value using GA1 and GA2 is represented by the solid and dashed lines, respectively, in Fig. 4. As in example 1, GA1 is not able to find the real maximum of fitness value but oscillates in the neighborhood, whereas GA2 reaches the real maximum in several hundred generations. It takes 497 generations for GA2 to find the real maximum. Therefore, $H_a = 49{,}700$ whereas $H_b = \binom{321}{15} = 2.712 \times 10^{25}$. Once again, GA2 is very efficient. Comparing examples 1 and 2, it is found that as the number of candidate locations increases, H_b increases exponentially, whereas H_a simply increases linearly. The efficiency of GA2 thus increases as the size of the candidate sensor set grows. In Fig. 5, the 15 optimal sensor locations selected using GA2 are illustrated.

Table 3 Frequencies and descriptions of target modes for space station photovoltaic array, example 2

Mode	Frequency, Hz	Description
1	0.112	Antisymmetric blanket first-order bending
2	0.114	Symmetric blanket torsion
3	0.123	Symmetric blanket first-order bending
4	0.133	Antisymmetric blanket first-order bending
5	0.155	Mast in-plane first-order bending
6	0.194	Symmetric blanket second-order bending
10	0.288	Symmetric blanket second-order bending/torsion

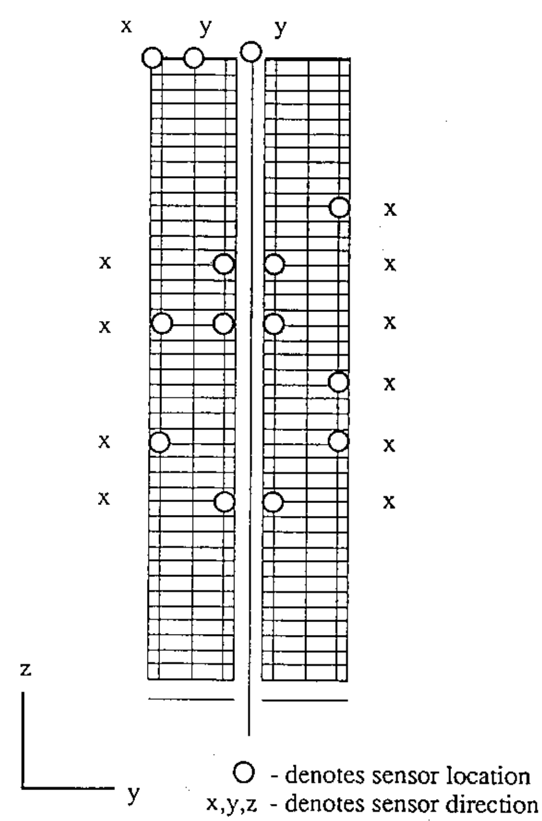

Fig. 5 Fifteen optimum sensor locations selected by GA2 algorithm for identification of 7 PV array target modes, example 2. Note that the location at the upper left corner represents two x direction sensors on opposite ends of a spring.

Table 4 Comparison of simulation results using GA2 and EfI for photovoltaic array, example 2

	15 sensors		30 sensors	
	Largest fitness value	No. of sensor locations different	Largest fitness value	No. of sensor locations different
GA2	8.7122e5	2	1.0465e8	3
EfI	8.6024e5		1.0363e8	

The GA2 and EfI methods are also compared for this example. Each algorithm was used to select the best 30, and then the best 15, sensor locations for the identification of the target modes. The results are compared in Table 4. Again, in each case it is seen that the sensor locations obtained by GA2 correspond to larger fitness values. To find 30 and 15 sensor locations, the running time on an IBM 386 PC compatible with coprocessor is 8400 and 3479 s, respectively, for GA2, compared with 277 and 281 s, respectively, for the EfI method.

Conclusion

The genetic algorithm with modified parent selection and children reproduction schemes has been successfully applied to the sensor placement problem for modal identification of an LSS. A feature called forced mutation is included to improve the convergence of the fitness value. By applying both the GA and the EfI method to sensor placement for an early version of the space station and a corresponding PV array, it has been shown that sensor configurations based on the GA are able to estimate the target mode response with higher accuracy, i.e., corresponding to a smaller confidence region. As the dimension of the FEM grows, the simulation results also show that there can be more sensor locations estimated by the EfI method that differ from the ones estimated by the GA. Moreover, the GA with modified selection and reproduction schemes is shown to be not only an efficient optimization algorithm compared to the exhaustive search techniques, but also more efficient than the original algorithm proposed in Refs. 10 and 11, at least for this problem.

In this paper, the performance index for the GA has been the determinant of the Fisher information matrix. In fact, there have been some papers using the trace or condition number of the FIM as the performance index of their proposed optimization schemes.[9,10] The GA can be easily modified to use any of these performance indices or even combinations of them with appropriately assigned weightings. The measurement noise at the sensor outputs has been assumed to be zero-mean Gaussian, uncorrelated between sensors, and constant in variance with respect to sensor location. The GA can also be easily applied to the case that the measurement noise is zero-mean Gaussian with arbitrary correlation matrix by using the original definition of the Fisher information matrix given in Eq. (10).

Compared with the EfI method, the main disadvantage of the GA is its computational complexity. It takes significantly more computation time than the EfI method for the GA to estimate the best sensor locations. In addition, since the real maximum of the fitness value is unknown a priori, the GA does not know when to stop, i.e., it does not know if the real maximum has been reached. One way of compensating for this is to set the no-improvement tolerance and to stop the GA if the fitness value has not been improved for a period larger than this tolerance. However, if the tolerance is too small, the GA will stop before the real maximum is reached, whereas if the tolerance is too large, a great deal of computation would be wasted after the real maximum is reached.

Appendix

The proof of Theorem starts with following lemma.

Lemma: Let $C \in R^{n\times m}$, $D \in R^{m \times n}$, and I_p be a $p \times p$ identity matrix; then

$$\det(I_n - CD) = \det(I_m - DC) \tag{A1}$$

Proof: See the Appendix in Ref. 21.

Proof of Theorem 1:

$$\det(B) = \det(A - r_i^T r_i)$$
$$= \det[A(I - A^{-1} r_i^T r_i)] \tag{A2}$$

Since A and $I - A^{-1} r_i^T r_i$ are both square matrices,

$$\det(B) = \det(A)\det(I - A^{-1} r_i^T r_i)$$
$$= \det(A)\det(1 - r_i A^{-1} r_i^T)$$
$$= \det(A)(1 - E_{Di}) \tag{A3}$$

where the previous lemma is used and $E_{Di} = r_i A^{-1} r_i^T$.

Recall that

$$B = A - r_i^T r_i = \sum_{j \neq i}^{n} r_j^T r_j = \Gamma_i^T \Gamma_i \tag{A4}$$

where $\Gamma_i = [r_1^T, \ldots, r_{i-1}^T, r_{i+1}^T, \ldots, r_n^T]^T$.

Since matrix B is still in factored form, B is positive semidefinite. This implies that $\det(B) \geq 0$ and thus $E_{Di} \leq 1$. It is assumed that A is positive definite, so A^{-1} is also positive definite. In other words, $\forall r_i \neq 0$, $i = 1, \ldots, n$,

$$\exists E_{Di} = r_i A^{-1} r_i^T > 0 \tag{A5}$$

However, r_i could be a zero row vector in Φ; therefore $E_{Di} \geq 0$.

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