Feasibility Report: Tuning an Inharmonic Bell

Associate Professor Neil McLachlan
mcln@unimelb.edu.au

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Brief
To determine whether it will be possible to design a bell with the first 5 overtones tuned to the frequency ratios described in Table 2.

Method
To take an existing harmonic bell profile and use *ReShape* Finite Element Analysis software to alter the bell profile through iterative shape optimisation.

At this preliminary stage, a free trial version that is limited to 350 elements was used. This results in a course FEA mesh that will provide indicative results only. A half model of the bell was used to reduce the number of elements in the model by exploiting the symmetry of the modal shapes.

Results
Figure 1 shows the geometry and FEA mesh of the last bell model.

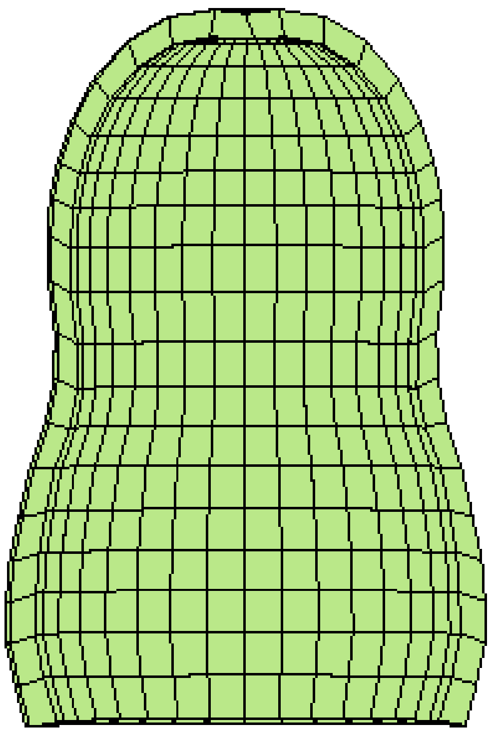
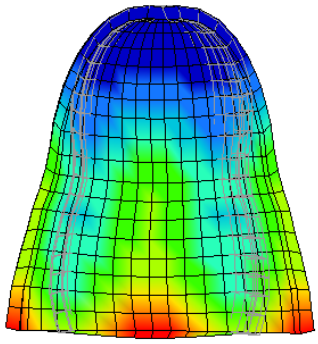
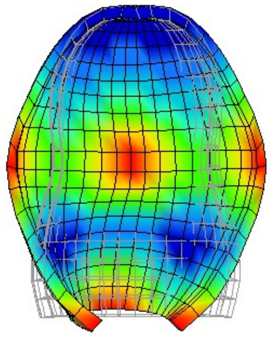
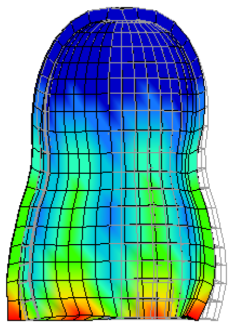
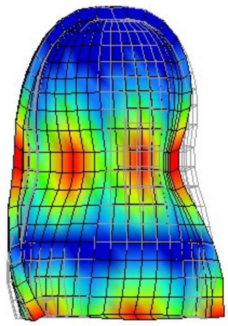
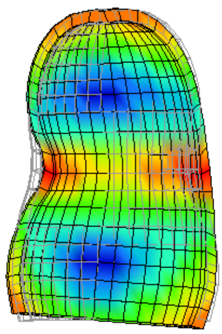
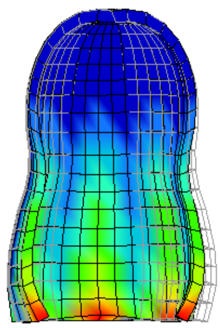


Figure 1. FEA mesh of the last bell model. Height = 283mm, Maximum diameter = 195mm, Mass = 11 kg in bronze.

Table 1. Mode Shapes

 2,0 mode	 2,1 mode
 3,0 mode	 3,1 mode
 0,2 mode	 4,0 mode

Explanatory notes on modes

The first number refers to the number of modal lines that pass through the centre of the bell (see Figure 2 below). The second number refers to the number of horizontal modal lines. Modes with no horizontal lines are the purely circumferential modes and tend to remain roughly equally spaced in frequency with shape changes (although they may be expanded or compressed). Modes with only horizontal lines (the 0,2 mode) are purely cylindrical. This mode is not usually reported for bells, as it doesn't occur when the bell head is fixed. Other modes are mixed. The frequencies of circumferential modes are increased by increasing circumferential stiffness and the frequencies of cylindrical modes by cylindrical stiffness. Mass loading at the maxima of modes decreases their frequency.

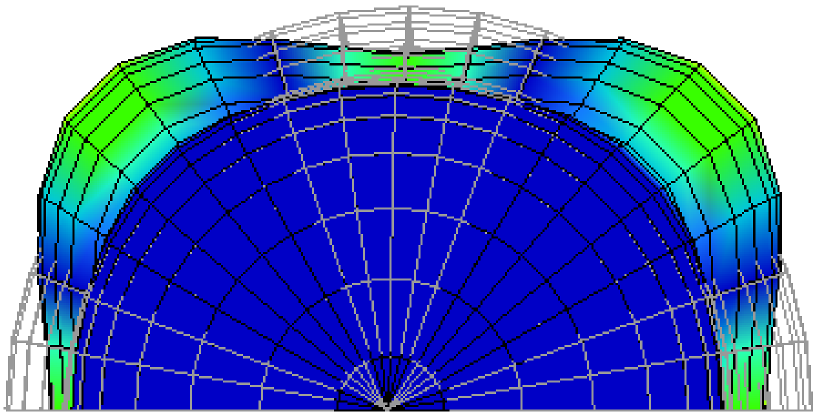


Figure 2. Plan view of the 4,0 mode showing 4 nodal lines passing through the centre of the bell.

Table 2. Frequencies and frequency ratios obtained in last 2 optimisation runs using this general profile.

Mode Type	Frequency (last)	Ratios (2nd last)	Ratios (last)	Ratio Targets
2,0	661	1	1	1
2,1	1313	1.60	1.99	1.79
3,0	1855	2.27	2.81	2.84
3,1	2239	3.87	3.39	4.02
0,2	3153	NA	NA	NA
4,0	3378	4.09	5.11	5.06

Table 2 shows optimisation results from the last two optimisation runs. Each run included 300 iterations of the FEA model. The 2nd last run obtained the required mode order, and the last run increased the ratios of the purely circumferential modes to close to the required ratios. Unfortunately this had opposite effects on the two mixed mode ratios (2,1 and 3,1) – in that one went up and the other down.

The very large changes in tunings possible for bells with this mode order suggests that with careful revision of the starting shape, the required modal tuning could be achieved. For example, the 3,1 mode is much more sensitive to changes in diameter than the 2,1 mode at the height of its upper maxima. There was no point attempting to fine-tune the circumferential modes using this course FEA mesh.

Conclusions

With careful choice of the starting geometry based on these observations, it is very likely that a bell with the required tuning could be achieved. This starting shape was found to be quite different from a harmonic bell in which the lower order tuned modes are all purely circumferential. In bronze, a bell with this general shape and a fundamental frequency of 65 Hz would weigh around 11.6 metric tonnes. A bell at 130 Hz would weigh just 1/8th of this, or around 1.5 metric tonnes. Using aluminium won't bring the mass down for the same fundamental as it produces frequencies about 1.5 times greater than bronze for the same size bell. So, although it's much lighter, the bell itself will need to be much bigger to have a 65 Hz fundamental.