RECURSIVE BLIND IMAGE DECONVOLUTION VIA DISPERSION MINIMIZATION

Cabir Vural and William A. Sethares

University of Wisconsin-Madison
Electrical and Computer Engineering Department
vural@cae.wisc.edu, sethares@engr.wisc.edu

ABSTRACT

This paper presents a method that uses an autoregressive filter for deblurring noisy blurred images blindly. The approach has several important advantages over using a finite impulse response filter. The optimum support of the adaptive autoregressive filter is the same as the support of the blur, and so the truncation error introduced by the finite support of the adaptive finite impulse response filter can be made arbitrarily small. Furthermore, the method can also be used for blur identification. In addition, resulting improvement in signal-to-noise ratios are higher and convergence of the adaptive filter coefficients is faster for a given blur. First, an autoregressive method is naively derived via a gradient method to minimize the dispersion. This leads to a recursion within a recursion which is computationally complex. Next, a simplification of the method is proposed. Finally, simulations demonstrate performance of the simplified method.

1. INTRODUCTION

A recorded image is usually a degraded version of the original because physical imaging systems are not perfect. Blur and observation noise are the most common degradations seen in recorded images, and often are unavoidable. The central problem in the field of image restoration is to reconstruct an unobservable *true image* from an observed *degraded image*.

If the blur (which is often called the Point Spread Function (PSF) in the literature) is assumed to be a Linear Shift Invariant (LSI) system, an observed image can be written (ignoring observation noise) as the Two-Dimensional (2-D) convolution of the true image with the blur. Restoration of the true image in the case of a known blur has been studied extensively giving rise to a variety of solutions [1] [2]. However, the blur is unknown in many practical cases. Hence, restoration of the true image must be performed from the degraded image alone, and this is called *blind image restoration or (deconvolution)*.

A modern comprehensive survey of existing blind image deconvolution methods can be found in the papers by Kundur and Hatzinakos [3] [4] according to which blind image deconvolution methods can be divided into two major groups: i) those which estimate the PSF *a priori* independent of the true image so as to use it later with one of the linear image restoration methods, and ii) those which estimate the PSF and the true image simultaneously. Algorithms belonging to the first class tend to be computationally simple, but they are limited to situations in which the PSF has a special parametric form, and the true image has certain features. Algorithms belonging to the second class, which are usually computationally more complex, must be used for more general situations.

A computationally simple blind image deconvolution method that is applicable to minimum or mixed phase blurs was presented in [5] and its convergence analysis was performed in [6]. The method is essentially a 2-D version of the Constant Modulus Algorithm (CMA) [7] [8] that is commonly used in the field of communications for blind equalization. The reader is referred to [9] and the references therein for a detailed introduction to the CMA and its analysis in the context of One-Dimensional (1-D) adaptive equalization.

A 2-D adaptive Finite Impulse Response (FIR) filter was used in [5]. The purpose of this paper is to present an analogous method that uses an adaptive 2-D Autoregressive (AR) filter for deconvolution. This approach has several important features. First, the analysis of the FIR implementation has shown that given a step size and a PSF, there is an optimum support for the FIR filter that must be determined experimentally. When using an AR deconvolution filter the optimum support is the same as the support of the blur. Hence, the distortion introduced by the finite support of the adaptive FIR filter can be made arbitrarily small. Furthermore, the FIR filter provides an approximate inverse to the blur at convergence. The AR filter converges to an approximation of the blur itself. Hence, the method can also be used for blur identification. In addition, resulting improvement in signal-to-noise ratios are higher and convergence of the adaptive filter coefficients is faster compared to the FIR case.

The FIR implementation is straightforward whether the adaptive filter has a causal support or not. The recursive implementation is not trivial if the adaptive filter does not have a causal support since 2-D AR filters having non-causal support are not recursively implementable. For implementation simplicity, this paper focuses on 2-D AR filters and FIR blurs with causal supports. Because 2-D non-causal filters can be decomposed into four 2-D causal filters [10], the results can be extended to the non-causal case with a suitable increase in complexity.

The organization of the paper is as follows. The blind image deconvolution problem for a spatially causal blur

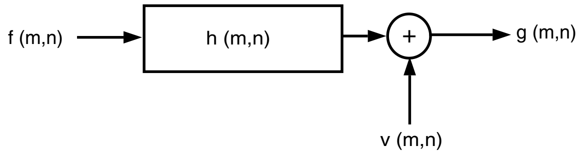

Fig. 1. Linear image degradation model.

is formulated in section 2. The algorithm for the recursive case is derived in section 3, which also discusses the Constant Modulus (CM) cost. Computer simulation results are provided in section 4. Section 5 concludes the paper.

2. PROBLEM FORMULATION

A model that describes the relationship between the unobservable true image and the observed degraded image is required by all blind image deconvolution algorithms. In general, blurs are assumed to be linear, though they may be shift-invariant or shift-variant. Similarly, the observation noise may be modeled as multiplicative or additive. This paper assumes a shift-invariant blur and additive Gaussian observation noise. Hence, the observed $M \times N$ degraded image $g(m,n)$ is given by

$$\begin{aligned} g(m,n) &= f(m,n) * h(m,n) + v(m,n) \\ &= \sum_{k=0}^{A-1} \sum_{l=0}^{B-1} h(k,l) f(m-k, n-l) + v(m,n) \end{aligned}$$

for $m = 0, \ldots, M-1$, $n = 0, \ldots, N-1$, where $h(0,0) = 1$ and $f(m,n)$, $h(m,n)$, $v(m,n)$ and $[0, A-1] \times [0, B-1]$ represent the true image, the PSF of the degrading system, additive noise that is independent of the true image and the support of the PSF, respectively. The linear image degradation model is depicted in Fig. 1. In blind image restoration, the PSF $h(m,n)$ is unknown. Therefore, the true image $f(m,n)$ must be estimated directly from the degraded image $g(m,n)$. While the values of the pixels of the true image are unknown, certain statistical properties are known; typically pixel values must be one of a small number of possibilities. As shown in [5], ambiguities in both gain and delay are inherent to blind image deconvolution. Keeping these ambiguities in mind, the blind image deconvolution problem can be stated as follows: *Obtain an estimate of the form* $\hat{f}(m,n) \approx \alpha f(m-m_0, n-n_0)$ *for some real* $\alpha \neq 0$ *and for some integers* m_0, n_0 *when only the observed image* $g(m,n)$ *is measurable. Both the true image* $f(m,n)$ *and the PSF* $h(m,n)$ *are assumed unknown.*

3. RECURSIVE IMAGE DECONVOLUTION VIA DISPERSION MINIMIZATION

The algorithm for the recursive case will be explained in detail in this section. Unless otherwise stated, pixel values of the true image are assumed odd integer-valued, i.e., pixel values may be $\pm 1, \pm 2, \ldots, \pm L-1$, where L is the number of gray levels in the true image. Many real images are 8-bit having 256 gray levels between 0 and 255. These images can be transformed to have odd-integer-valued gray levels by a uniform or non-uniform thresholding based on the probability density function of the true image. The CM cost will be studied first to set the stage for the recursive blind algorithm.

3.1. The CM Cost

Even though traditional uses of the CM cost have all 1-D, the CM cost can be extended for use in 2-D. The CM cost term was introduced for blind equalization of communication signals over dispersive channels by Godard [7] and Treichler and Agee [8]. This section generalizes the CM cost for use in 2-D by reformulating the cost for a real-valued zero-mean true image $f(m,n)$ and a real-valued PSF $h(m,n)$. It is assumed that each gray level of the true image is equally likely (a suitable preprocessing of the degraded image such as histogram equalization may be required to satisfy this condition). The CM cost is given by

$$J_{CM} := E[(\hat{f}^2(m,n) - \gamma)^2] \tag{1}$$

where

$$\gamma := \frac{E[f^4(m,n)]}{E[f^2(m,n)]} \tag{2}$$

is the *dispersion constant* of the true image. It is evident from Eq. (1) that the CM cost penalizes the deviations (or dispersion) of $\hat{f}^2(m,n)$ from constant γ, which is why the method in [5] was called *blind image deconvolution via dispersion minimization*.

Plotting the CM cost versus the adaptive filter parameters results in a surface called the *CM cost surface*. The method of blind image deconvolution via dispersion minimization attempts to minimize the CM cost by starting at some location on the surface and following the trajectory of steepest descent.

3.2. Proposed Method

Fig. 2 illustrates the recursive blind image deconvolution via dispersion minimization method, where the degraded image $g(m,n)$ is applied to an adaptive AR filter whose purpose is to estimate the true image $f(m,n)$. Since the true image is unknown, a desired image (the true image in the ideal case) must be generated artificially from the estimated true image $\hat{f}(m,n)$. The function of the zero-memory nonlinearity (the rightmost term in Fig. 2) is to generate an "artificial" image $\hat{f}_{NL}(m,n)$ so that an error term $e(m,n)$ that drives the recursive algorithm can be obtained. The zero-memory nonlinearity is chosen such that the error term $e(m,n)$ corresponds to the negative gradient of J_{CM}.

Transforming the 2-D signals and filters to the corresponding 1-D signals and filters using appropriate index mappings is useful to simplify the derivation of the recursive algorithm.Observe that a 2-D filter $w(m,n)$ with support $[0, A-1] \times [0, B-1]$ can be transformed to a 1-D filter $w(k)$ by the "lexicographic ordering" $T_1 : R_2 \rightarrow R_1$ such that $k = mB + n$, where

$$R_2 = \{(m,n) |\; 0 \leq m \leq A-1,\;\; 0 \leq n \leq B-1\} \tag{3}$$

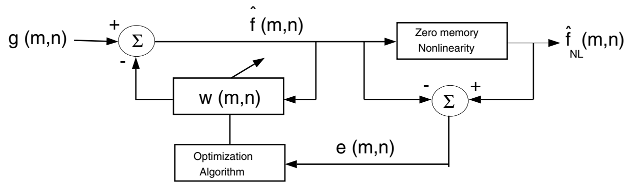

Fig. 2. Block diagram of recursive blind image deconvolution via dispersion minimization.

$$R_1 = \{k|\ 0 \le k \le S_1 S_2 - 1\}\,. \tag{4}$$

Similarly, a 2-D signal $f(m,n)$ can be transformed to a 1-D signal $f(k)$ by the "local lexicographical ordering of support $[0, A-1] \times [0, B-1]$" $T_2 : P_2 \to P_1$ such that

$$P_2 = \{(r,s) \mid m - A \le r \le m,\ \ n - B \le s \le n\} \tag{5}$$

$$P_1 = \{t \mid k - AB + 1 \le t \le k\} \tag{6}$$

where A, B are constants, $0 \le m \le M-1$, $0 \le n \le N-1$, and $k = T_2(m,n)$ is a suitable function of (m,n). The output of the AR filter at the jth iteration for the (m,n)th pixel $\hat{f}_j(m,n)$ is an estimate of the true image given by

$$\hat{f}_j(m,n) = g(m,n) - \sum_{r=0}^{A-1}\sum_{s=0}^{B-1} w_j(r,s)\hat{f}_j(m-r, n-s) \tag{7}$$

where $(r,s) \neq (0,0)$. This estimate can be rewritten by using mappings T_1 and T_2 as

$$\hat{f}_j(k) = \sum_{i=1}^{AB-1} w_j(i)\hat{f}_j(k-i) \tag{8}$$

where $g(k)$, $\hat{f}_j(k)$ and $w_j(i)$ are the 1-D representation of the degraded image, the output of the AR filter and the adaptive filter coefficients at the jth iteration resulting from applying the index mappings T_1, T_2 to their 2-D counterparts (note that j is the time iteration variable, while k is the spatial position).At the beginning, the adaptive filter is far from being a reliable estimate of the blur. Hence, the estimate $\hat{f}_j(k)$ is not reliable,though it may be used in an adaptive scheme to obtain a better estimate for the next pixel by minimizing the CM (dispersion) cost. Gradient Descent (GD) methods are generally used to solve for CM estimators because closed form expressions do not usually exist. Since exact GD requires statistical knowledge of the degraded image that is unavailable in real applications, stochastic GD method are utilized. The general form of the recursive stochastic GD algorithm for minimizing the CM cost is

$$w_{j+1}(l) = w_j(l) - \mu\frac{dJ_{CM}}{dw_j(l)},\ \ l = 1, \ldots, AB-1 \tag{9}$$

where μ is a small positive step-size. Because it is not possible to minimize an expected value directly, the method uses an instantaneous estimate J of J_{CM} defined as

$$J := \frac{1}{4}(\hat{f}_j^2(k) - \gamma)^2. \tag{10}$$

Therefore, for the kth pixel coefficients of the adaptive filter are updated according to

$$w_{j+1}(l) = w_j(l) - \mu\frac{dJ}{dw_j(l)}. \tag{11}$$

As shown in [11], Eq. (11) can be written as

$$w_{j+1}(l) = w_j(l) + \mu\left(\hat{f}_j^2(k) - \gamma\right)\hat{f}_j(k)\varphi_{j,l}(k). \tag{12}$$

where

$$\varphi_{j,l}(k) = \hat{f}_j(k-l) - \sum_{\substack{i=1 \\ i \neq l}}^{AB-1} w_j(i)\varphi_{j,l}(k-i) \tag{13}$$

is called *regressor filtering*. Eq. (12) can be vectorized as

$$\mathbf{w}_{j+1} = \mathbf{w}_j + \mu\left(\hat{f}_j^2(k) - \gamma\right)\hat{f}_j(k)\boldsymbol{\varphi}_j(k) \tag{14}$$

where $\mathbf{w}_j$ and $\boldsymbol{\varphi}_j(k)$ are the lexicographically ordered adaptive filter parameter vector at the jth iteration and the regressor filter vector for the kth position given by

$$\mathbf{w}_j := [w_j(1), w_j(2), \ldots, w_j(AB-1)]^T \tag{15}$$

$$\boldsymbol{\varphi}_j(k) := [\varphi_{j,1}(k), \varphi_{j,2}(k), \ldots, \varphi_{j,AB-1}(k)]^T. \tag{16}$$

Regressor filtering defined in Eq. (13) makes implementation of the recursive algorithm costly. A simplified algorithm that bypasses the regressor filtering would be preferred. An approximate gradient for the recursive case uses the currently available data vector in place of the regressor filtered version, that is,

$$\boldsymbol{\varphi}_j(k) \approx [\hat{f}_j(k-1), \ldots, \hat{f}_j(k-AB+1)]^T. \tag{17}$$

Equations (14) and (17) together with (7) constitute the *recursive blind image deconvolution via dispersion minimization* algorithm. The output of the adaptive AR filter $\hat{f}(k)$ is an estimate of the true image $f(k)$, and the coefficients $w(k)$ provide an estimate of the PSF $h(m,n)$ at convergence.

4. EXPERIMENTAL RESULTS

The classical 8-bit gray-scale *pepper* image was chosen as a test image. Histogram equalization was performed on the test image which results in approximately uniformly distributed image. Then, its mean was subtracted from the histogram equalized image to get a zero-mean image. Finally, a uniform quantization was applied to the uniformly distributed image to obtain a 4-bit true image.

Observe that the CM cost is non-convex. Hence, the method proposed may converge to a local minimum instead of the global minimum of J_{CM} depending on how it is initialized. If there is no *a priori* information about the PSF, the adaptive filter is initialized using zero values for all coefficients. If there is *a priori* information about the PSF, this information may aid in initializing the adaptive filter in a better way.

Fig. 3 depicts the 4-bit true image (left), degraded image (middle) and estimated true image (right), from

Fig. 3. Deconvolution result for a 4-bit image. (left) True image; (middle) Degraded image; (right) Estimated true image.

h(m,n)	n: 0	1	2	3	4
m: 0	1	.7155	.3536	.1707	.0894
1	.7155	.5443	.2963	.1527	.0831
2	.3536	.2963	.1925	.1141	.0680
3	.1707	.1527	.1141	.0775	.0512
4	.0894	.0831	.0680	.0512	.0370

Table 1. The PSF used to obtain the blurred image.

w(m,n)	n: 0	1	2	3	4
m: 0	1	.7278	.3611	.1733	.0554
1	.7239	.5469	.3208	.1834	.0864
2	.3916	.3251	.2096	.1428	.0754
3	.1736	.1733	.1117	.0699	.0151
4	.0287	.0716	.0351	.0013	-.0537

Table 2. The adaptive filter coefficients at convergence.

which it is clear that the method is useful in deblurring the degraded image. Table 1 provides the PSF used. Table 2 shows coefficients of the adaptive filter at convergence. It is obvious from Table 2 that the adaptive filter converges to the PSF well except for a few coefficients.

5. CONCLUSIONS

This paper presented a recursive method for blind image deconvolution that uses an adaptive autoregressive filter. As was seen, the presence of regressor filtering in a true gradient makes the recursive implementation computationally costly. A simplified algorithm that bypasses the regressor filtering was proposed. The simplified method was shown to work through a computer simulation. The simplified recursive algorithm is not a stochastic gradient descent algorithm because of the removal of the regressor filtering. Consequently, it is important to study its behavior to find conditions on the PSF under which the algorithm converges to a desirable solution . A sufficient condition in the absence of observation noise for the special case when the true image is binary is that the PSF satisfy the "Strictly Positive Real" (SPR) condition [11]. Common PSFs frequently encountered in practice may or may not satisfy this condition. If a PSF does not satisfy the SPR condition, one has to implement the recursive method without ignoring the regressor filtering so as to guarantee its stability.

6. REFERENCES

[1] M. I. Sezan and A. M. Tekalp, "Survey of recent developments in digital image restoration," *Optical Engineering*, vol. 29, no. 5, pp. 393–404, May 1990.

[2] M. A. Bahnam and A. K. Katsaggelos, "Digital image restoration," *Signal Processing Magazine*, vol. 14, no. 2, pp. 24–41, March 1997.

[3] D. Kundur and D. Hatzinakos, "Blind image deconvolution," *IEEE Signal Processing Magazine*, vol. 13, no. 3, pp. 43–64, May 1996.

[4] D. Kundur and D. Hatzinakos, "Blind image deconvolution revisited," *IEEE Signal Processing Magazine*, vol. 13, no. 6, pp. 61–63, Nov. 1996.

[5] C. Vural and W. A Sethares, "Blind image deconvolution via dispersion minimization," Submitted to IEEE Tran. Image Processing.

[6] C. Vural and W. A. Sethares, "Convergence analysis of blind image deconvolution via dispersion minimization," Submitted to IEEE Tran. Signal Processing.

[7] D. Godard, "Self-recovering equalization and carrier tracking in two dimensional data communication systems," *IEEE Trans. Commun.*, vol. 28, no. 11, pp. 1867–1875, Nov. 1980.

[8] J. R. Treichler and B. G. Agee, "A new approach to multipath correction of constant modulus signals," *IEEE Trans. Commun.*, vol. 31, no. 2, pp. 459–473, Apr. 1983.

[9] J. R. Johnson et al., "Blind equalization using the constant modulus criterion: A review," *Proceedings of the IEEE*, vol. 86, no. 10, pp. 1927–1950, Oct. 1998.

[10] D. E. Dudgeon and R. M. Merserau, *Multidimensional digital signal processing*, Prentice Hall, Englewood Cliffs, New Jersey, 1984.

[11] Cabir Vural, *Blind Image Deconvolution Via Dispersion Minimization*, Ph.D. thesis, University of Wisconsin-Madison, 2002.