Bursting in Adaptive Hybrids

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Abstract—**Adaptive filtering techniques have been successfully applied to attenuate echoes in long distance telephone connections. Recently, with the increased ability to process signals cheaply, adaptive echo cancellation is being employed on shorter telephone circuits. While echo cancelers do tend to be effective on the shorter circuits, a new (and undesirable) phenomena called bursting has been observed. Bursting is characterized by long periods of successful echo attenuation alternating with short periods of wildly oscillating signals. This paper studies bursting by constructing a pair of simplified models of adaptive hybrid systems. The models are analyzed when excited by various dc and sinusoidal inputs, and the results are related back to the systems of interest, providing insight into the fundamental sources of the bursting problem—an imbalance of excitation and the enclosure of an adaptive filter in a feedback loop. Simulations provide corroborating evidence.**

I. Introduction

"Echo loves Narcissus."—Grecian graffiti

THE fundamental problem addressed by echo control in telephone systems is illustrated in Fig. 1. Echoes originate in the four to two wire conversion inside the hybrid circuit. Inevitably, there is a mismatch in the impedance characteristics of the two-wire loop and the balancing network of the hybrid circuit, and some energy from the four wire received signal is returned in the four-wire transmitted signal. The application of adaptive filtering technology to remove a significant proportion of this echo has been so successful that it appears in recent introductory texts on adaptive filtering, e.g., [1] and [2], as prime evidence of the utility of adaptive filtering. An adaptive hybrid incorporating echo cancellation is illustrated in Fig. 2.

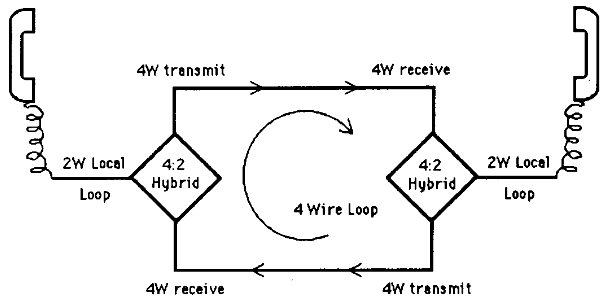

Fig. 1. The echo problem in telephone systems.

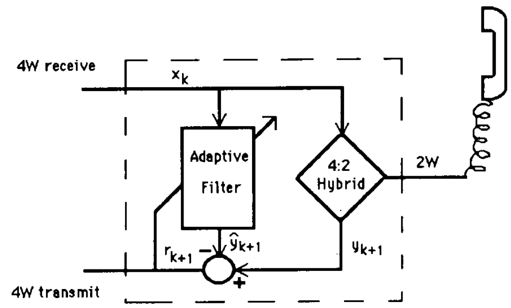

Fig. 2. Basic adaptive hybrid.

The generation of the echo can be modeled by passing the received signal x_k through a linear time invariant filter with a finite impulse response h as in Fig. 3. Two signals are added to the echo before the sum y_k is returned to the adaptive echo canceler. A "noise" signal η_k encompasses the part of the echo which cannot be captured by the FIR filter representation, noise due to quantization in digital implementations, and other small disturbances. The signal v_k represents the speech signal of the near end speaker which should be allowed to pass through the hybrid and canceler undisturbed. The adaptive hybrid attempts to remove the part of y_k which is significantly correlated with x_k within the time windows set by the length of the adaptive FIR filter.

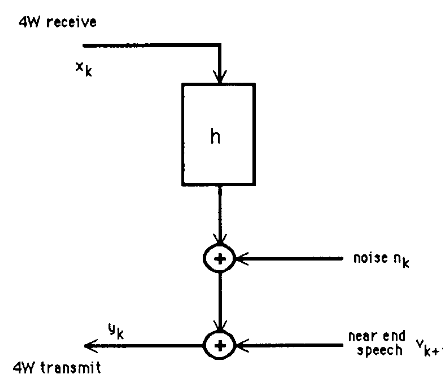

Fig. 3. Model of the 4:2 hybrid (echo path).

Existing theory suggest certain operating conditions under which such an adaptive echo canceler should perform well. Theoretical investigations of adaptive filters demonstrate desirable behavior (without their encasement in a feedback loop such as in Fig. 4) when driven by "persistently exciting" inputs [3] and when disturbances are small. Thus, the adaptive process benefits when the far-end input contains a frequency range of components well-distributed enough to persistently excite the adaptive system at the near end and the interfering signals v_k and η_k are small. These conditions are fairly closely met in echo canceler implementations on long distance connections. For example, sufficiently rich excitation is usually provided by speech signals. In common practice, double talk detectors turn off adaptation when v_k is too large relative to x_k, and adaptive filter lengths are chosen (in part) to keep η_k small.

One shortcoming of the standard analysis of adaptive

Paper approved by the Editor for Channel Equalization of the IEEE Communication Society. Manuscript received October 15, 1987; revised April 12, 1988. This work was supported by the NSF under Grant DCI-8608787. This paper was presented in part at the 1988 IFAC Workshop on Robust Adaptive Control, August 1988.

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IEEE Log Number 8929094.

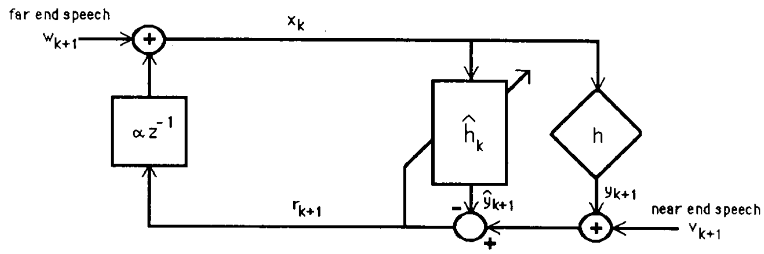

Fig. 4. Single adaptive hybrid.

systems when applied to this echo-cancellation problem is its reliance on decorrelation of the "inputs" from the "disturbances" [4]. In the echo cancellation context, this would require that x be generated independently from v. Yet the 4W receive (x) for the right-hand canceler of Fig. 1 invariably contains a component which represents the residual echo of its own 4W transmit (y) reflected through the hybrid on the left-hand side. Since the 4W transmit (y) includes the speech (v) at the right-hand canceler, there is a source of correlation between x and a sequentially correlated v, which is essentially due to a feedback of the transmitted signal. When the length of the delay introduced in the four-wire path is long enough and the signals themselves are broad-band enough, this source of correlation is benign, since it is outside the time window of the adaptive filter. The effects of the correlation, however, become more pronounced as the length of the connection (and hence the length of the delay)is decreased.

In an effort to further automate echo control and to comply with prescribed echo loss requirements in connections to common carriers, adaptive filters such as the adaptive hybrid of Fig. 2 are being used in many short connections. Such short connections do not provide the decorrelation delay of long distance lines, so x and v can be significantly correlated, especially when v is present and w is not. Given the possibility of such situations, predictions of robust behavior cannot be based on a persistent excitation and small disturbance combination [5] or on a near-decorrelation of the excitation and disturbance [6]. One attempt to guarantee that the excitation is adequate while the disturbance remains small is to use a doubletalk detector which switches adaptation off (or on) when the ratio of the energy in v to the energy in x rises above (or falls below) a given threshold. There is, however, a danger in the aggressive use of doubletalk detectors. One special feature of the adaptive hybrid is that it operates in a closed, rather than in an open-loop mode. The simple hybrid model of Fig. 4 may be thought of as a feedback system with two inputs, τ and w, and two outputs, r and x. If v is large and the doubletalk detector halts adaptation at a fixed $\hat{h}$, the closed-loop system has the characteristic equation $1 - \alpha(h - \hat{h})z^{-1}$. If $|\alpha(h - \hat{h})| > 1$, the resulting time-invariant feedback loop is unstable. Imagine switching into a circuit with a nominal $\hat{h}$ but a worst case (α, h) pair. Conceivably, if the loop gain $|\alpha(h - \hat{h})| > 1$ and the doubletalk detector is immediately engaged by the message traffic, this initial instability will not be corrected before the circuit experiences sustained "singing." To avoid entrapment in a singing mode, doubletalk detectors should be used conservatively.

What happens if doubletalk detectors are removed and x is substantially correlated with v within the filter length window of the adaptive FIR echo canceler? A new phenomenon, called bursting, has been observed in experimental tests at Tellabs Research Laboratory specifically designed to examine such cases [7]. Long periods of close match between the output of the adaptive filter and the echo path during which the echo canceler appears to be functioning well, suddenly (and with no apparent warning) degenerate into wild oscillation, which then restabilizes just as suddenly. Real time laboratory tests at Tellabs utilized a 20 tap adaptive hybrid at the near end of a line and a simple (nonadaptive) hybrid at the far end. With independent narrow-band modem signals of approximately equal amplitude at each end, and with a nonadaptive echo attenuation of about -6 dB, such bursts appeared intermittently. When the transmission at the far end was quiescent, the bursts appeared more frequently. It is this latter case of extremely imbalanced excitation (with w zero and v nonzero) in conjunction with the absence of doubletalk detectors that we will exploit in order to explain this experimentally observed bursting.

This paper characterizes the bursting (mis)behavior as long periods of good echo attenuation (during which the parameters of the adaptive mechanism slowly drift towards a setting destabilizing the closed-loop system) alternating with brief periods of rapid oscillation of signals throughout the system (during which the adaptive mechanism quickly restabilizes). Figs. 6–8 show such bursting in simple simulations of adaptive hybrids.This bursting of the adaptive hybrid is closely related to "bursting" in adaptive control [8] in that it is the result of underexcitation and disturbances combined with the existence of a feedback path around the adaptive mechanism. The parameter drift of the quiescent phase to the parameter drift observed in the underexcited LMS adaptive filter in [9]. The recovery from the burst is closely related to the "self-stabilization" property of [10] in which the adaptive algorithm generates an exponentially growing output error (the burst) which it then uses as a form of spectrally rich, self-generated excitation in order to regain stability.

II. Problem Statement and Overview

Models for studying the salient features of adaptive echo cancelers on phone lines are diagrammed in Figs. 4 and 5. The first supposes that a single parameter adaptive hybrid is present at the near end of the line and that a simple (nonadaptive) hybrid introduces an echo (a scaled unit delay) at the far end. The second supposes that adaptive hybrids are present at both ends of the phone line.

The adjustment of the adaptive parameters, represented by the arrow through the $\hat{h}$ box in Fig. 4, uses the error $y_k - \hat{y}_k$ ("corrupted" by v_k) to adjust the $\hat{h}$ parameter with the LMS parameter update law

$$\hat{h}_{k+1} = \hat{h}_k + \mu x_k r_{k+1} \tag{2.1}$$

where μ is the stepsize,

$$r_{k+1} = y_{k+1} - \hat{y}_{k+1} + v_{k+1} \tag{2.2}$$

is the error sequence,

$$y_{k+1} = hx_k \tag{2.3}$$

is the near-end echo source,

$$\hat{y}_{k+1} = \hat{h}_k x_k \tag{2.4}$$

is the output of the adaptive filter, and

$$x_k = \alpha r_k + w_k \tag{2.5}$$

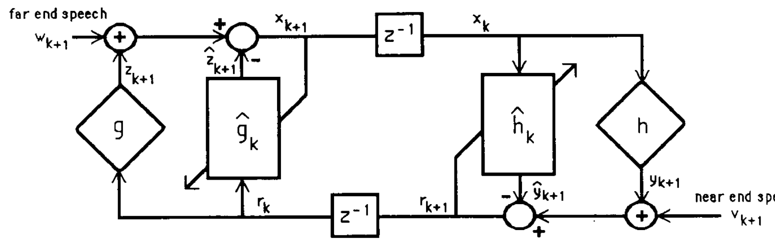

Fig. 5. Double adaptive hybrid.

is the echo from the far end. In (2.5), α is a small positive constant representing the attenuation of the echo and w_k is the signal transmitted from the far end. Combining (2.1) to (2.5) and introducing the parameter estimate error variable $\tilde{h}_k = h - \hat{h}_k$, produces

$$\tilde{h}_{k+1} = (1 - \mu x_k^2)\tilde{h}_k - \mu x_k v_{k+1} \tag{2.6}$$

$$x_{k+1} = \alpha \tilde{h}_k x_k + \alpha v_{k+1} + w_{k+1}. \tag{2.7}$$

Typical operation of the system supposes that the near and far end transmissions v_k and w_k alternate frequently, which helps to ensure that the algorithm is adequately excited. Indeed, if w_k and v_k are relatively uncorrelated and if w_k is persistent (so x_k is persistent), then (2.6) is exponentially stable and hence is convergent to some ball about $\tilde{h} = 0$ (which implies that $\hat{h}$ is nearly equal to h, and that the echo is suppressed). This is formalized in result 1 of Section 3.

The bursting phenomena appears when the transmission at the far end is quiescent while the transmission at the near end is active. This is first investigated under the simplifying assumption that $w_k = 0$ and $v_k = 1$ for every k. The parameters of the adaptive mechanism are shown to "wind up" or "drift" until they are eventually pushed across the stability boundary. Signal growth follows (this is the burst) and then the systems restabilizes. This situation is analyzed in result 2 and is simulated in Fig. 6, which shows large bursts after about 2300 iterations of good behavior. The analysis and simulations are then extended to consider the more realistic situation when v_k is sinusoidal, see result 3 and Fig. 7.

The second situation of interest is when there are adaptive hybrids at both the near and far ends of the phone connections as in Fig. 5, but the disparate excitation still occurs Defining the parameter estimate errors $\tilde{h}_k = h - \hat{h}_k$ and $\tilde{g}_k = g - \hat{g}_k$ and following the logic of (2.1) to (2.7) shows that the equations representing the system are

$$\tilde{h}_{k+1} = (1 - \mu x_k^2)\tilde{h}_k - \mu x_k v_{k+1} \tag{2.8}$$

$$\tilde{g}_{k+1} = (1 - \mu r_k^2)\tilde{g}_k - \mu r_k w_{k+1} \tag{2.9}$$

$$x_{k+1} = \tilde{g}_k r_k + w_{k+1} \tag{2.10}$$

$$r_{k+1} = \tilde{h}_x x_k + v_{k+1} \tag{2.11}$$

where $v_k(w_k)$ is the input at the near (far) end. When $\tilde{g}$ is small, the input at the far end provides excitation for the near end adaptive algorithm (2.8). When $\tilde{h}$ is small, the near-end input provides excitation for the far end algorithm (2.9). Paradoxically, the input at the near (far) end acts as a disturbance to the algorithm at the near (far) field.The results of Section IV show that when the adaptive filters are of sufficient order to exactly match the dynamics of the echo path, bursting will not occur. A more realistic scenario is to suppose that the complexity of the echo source is greater than the complexity of the adaptive mechanism. To model this situation, suppose that the echo source at the far end is

$$z_{k+1} = g_1 r_k + g_2 r_{k-1} \tag{2.12}$$

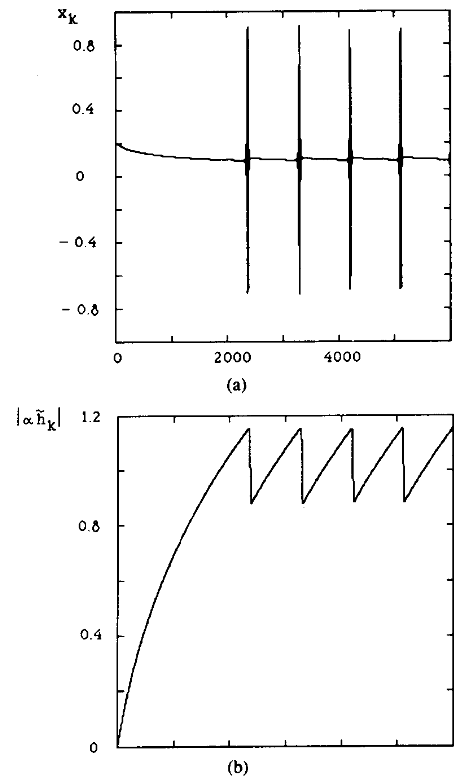

Fig. 6. Bursting in single end adaptive hybrid with constant near end transmission. (a) Received signal. (b) Pole location.

while the adaptive estimator has only a single adjustable parameter

$$\hat{z}_{k+1} = \hat{g}_k r_k. \tag{2.13}$$

Then (2.8) to (2.11) become

$$\tilde{h}_{k+1} = (1 - \mu x_k^2)\tilde{h}_k - \mu x_k v_{k+1} \tag{2.14}$$

$$\tilde{g}_{k+1} = (1 - \mu r_k^2)\tilde{g}_k - \mu g_2 r_k r_{k-1} \tag{2.15}$$

$$x_{k+1} = \tilde{g}_k r_k + g_2 r_{k-1} \tag{2.16}$$

$$r_{k+1} = \tilde{h}_k x_k + v_{k+1}. \tag{2.17}$$

where w_k has already been set to zero.

With $w_k = 0$ and v_k a sinusoid, r_k is "almost" a sinusoid. The parameter error at the far end rapidly converges to $\tilde{g} = -g_2$. The signal x_k then approaches $-r_k + r_{k-1}$ and $\tilde{h}$ becomes an integration of $r_{k+1}(-r_k + r_{k-1})$. Over a period, this is a small constant, and so $\tilde{h}$ is driven until the closed loop system becomes unstable. This is the burst. The analysis is formalized in Result 5.

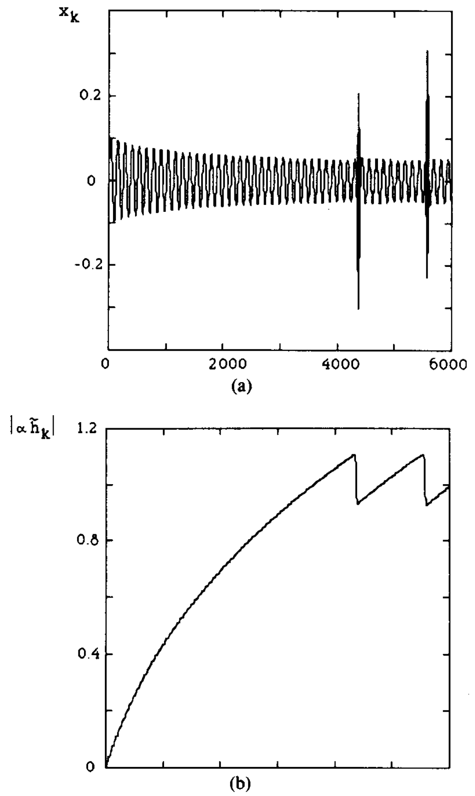

Fig. 7. Bursting in single ended adaptive hybrid with sinusoidal near-end transmission. (a) Received signal. (b) Pole location.

Thus, the undermodeled double active hybrid is susceptible to the same bursting as the single adaptive hybrid. There are two differences. First, bursting is an intrinsic property of the single adaptive hybrid, while it is result of mismodeling in the dual hybrid scenario. Second, the "integration" that leads to bursting in the single hybrid case is driven by a $\sin^2$, while the "integration" that causes bursting in the double hybrid scenario is dependent on the phase difference between r_k and r_{k-1}. Consequently, the drift of the parameters towards the bursting instability is much slower in the double than in the single hybrid situation. Similarly, smaller g_2 imply smaller x_k, and the drift is even slower. Whereas the bursts in simulations of the single adaptive hybrid tend to occur after a few thousand iterations, the simulations of the double hybrid case with mismodeling often do not burst until 45 000 iterations!

The practical consequence of the slower rate is that bursting is less likely to occur in a finite time frame. The possibility of bursting in real situations is thus reduced by close modeling of the echo path an by bandlimiting the inputs, (which reduces the phase difference that drives the drift) when there are adaptive hybrids at both ends of the connection.

III. The Single Adaptive Hybrid System

This section examines the stability properties of a single parameter adaptive echo canceler implemented at the near end of a phone line. As in Fig. 4, the echo source at the far end is modeled by a delayed and attenuated feedback of the transmitted signal. The "normal" situation is when the transmissions from both the near and far ends are persistent. Under fairly mild excitation conditions, the adaptive echo canceler is stable.

Result 1 (Stability of single adaptive hybrid with adequate excitation): Consider the system (2.6)–(2.7) and suppose that w_k is adequately excited (definition 2, Appendix A). If $\tilde{h}_0$ is small, then there exist v^* and μ^* such that for every sequence v_k bounded by v^*, and for every $\mu \in (0, \mu^*)$, $\tilde{h}_k$ converges to a ball of radius $\delta(v^*)$ about zero where $\delta(v^*) \to 0$ as $v^* \to 0$.

Proof: See Appendix B.

This result shows that when w_k is adequately excited, small v_k imply that $\hat{h}$ almost matches h, that is, that the echo is nearly canceled and the system remains stable. In particular, it is possible to guarantee that $|\alpha \tilde{h}_k| < 1$ for every k by choice of v^*, which implies that the bursting behavior described below cannot occur. The weakness of this result is that it gives no precise measure of the magnitudes involved. The following example explores this in one simple situation.

Example 1: Suppose $w_k = w > 0$ and $v_k = v > 0$ for all k. If $w > \alpha v$, then (2.6)–(2.7) has a locally stable equilibrium at $\tilde{h}^* = -v/w$ and $x^* = w$.

Proof: See Appendix B.

Thinking of w as the degree of excitation, and v as the power of the disturbance, this example suggests that the relation $w > \alpha v$ is somehow fundamental to the proper operation of the adaptive hybrid. Indeed, when the inequality is violated, then the "equilibrium" $\tilde{h}^* = -v/w$ causes $|\alpha \tilde{h}^*| = |\alpha v/w| > 1$, which implies that (2.7) is unstable. Such instability is at the heart of the bursting phenomena.

Bursting arises when the far end of the phone line is quiescent and the near end is active. The simplest way to model this situation is to suppose that $w_k = 0$ and $v_k = 1$ for every k (this certainly violates the hypothesis of Example 1). The system (2.6)–(2.7) then becomes

$$\tilde{h}_{k+1} = (1 - \mu x_k^2)\tilde{h}_k - \mu x_k \tag{3.1}$$

$$x_{k+1} = \alpha \tilde{h}_k x_k + \alpha, \tag{3.2}$$

which has no finite stationary points. Solving (3.1) for $\tilde{h}^* = \tilde{h}_{k+1} = \tilde{h}_k$ yields $\tilde{h}^* = -1/x^*$. Plugging this into (3.2) gives $x^* = 0$. Thus, as x converges towards its "equilibrium" at 0, $\tilde{h}$ tries to converge to its "equilibrium" at ∞. Eventually, $\tilde{h}$ grows large enough to destablize (3.2). This is one interpretation of the origin of the bursting phenomenon.

Equations (3.1) and (3.2) are easily simulated. The values of parameters in the simulation of Fig. 6 were chosen conservatively. The echo attenuation factor is $\alpha = 0.2$, the echo path at the near end is $h = 0.1$, and the estimator was initialized at $\hat{h}_0 = 0$ with a stepsize of $\mu = 2^{-5}$. Fig. 6(a) shows the received signal x_k versus time, while Fig. 6(b) plots $|\alpha \tilde{h}_k|$ versus time. The quantity $\alpha \tilde{h}_k$, for satisfactorily slow $\tilde{h}_k$, can be interpreted as the "instantaneous" pole of (3.2). The first burst occurs shortly after 2300 iterations, and then the bursts recur approximately every 600 iterations. Though the qualitative features of the simulations are easily reproducible, the exact "frequency" and "magnitude" of the bursts appears to be fairly sensitive to initial conditions, round-off errors, and quantization errors.

The bursting cycle consists of the following.

1) A long linear drift of $\tilde{h}$ driven by the competing rate phenomenon as in [9], coupled with a slow decay of x which cause $|\alpha \tilde{h}|$ to grow larger than 1 (at time t_1).

2) This instability causes x to expand rapidly, until x is larger than α at time t_2. The contraction term of the competing rate lemma dominates, and $\tilde{h}$ begins to shrink. This is essentially the self stabilization of [10].

3) At time t_3, $|\alpha \tilde{h}|$ becomes less than unity, restabilizing (3.2), and finally,

4) x decays, until at time t_4, the situation has returned to step 1).

Due to the (relative) simplicity of (3.1)–(3.2), this bursting cycle can actually be proven.

Result 2 (Bursting with constant disturbance): Consider the system (3.1) and (3.2). Suppose there are constants ϵ and t_0

with $\alpha/2 \gg \epsilon > 0$ such that

i) $\epsilon < x_{t_0} < \alpha - \epsilon$,
ii) $|\alpha\tilde{h}_{t_0}| < 1 - \epsilon$, and
iii) $\tilde{h}_{t_0} < 0$.

Then there is a μ^* such that for every $\mu \in (0, \mu^*)$, there exist $t_1 < t_2 < t_3 < t_4 < \infty$ such that

1) $|\alpha\tilde{h}_{t_1}| > 1 - \epsilon$, $x_{t_1} < \alpha - \epsilon$ (the drift),
2) $|\alpha\tilde{h}_{t_2}| > 1 + \epsilon$, $x_{t_2} > \alpha - \epsilon$ (the push across the stability boundary),
3) $|\alpha\tilde{h}_{t_3}| < 1 - \epsilon$, $x_{t_3} \gg \alpha$ (the burst), and
4) $|\alpha\tilde{h}_{t_4}| < 1 - \epsilon$, $\epsilon < x_{t_4} < \alpha - \epsilon$ (restabilization).

Proof: See Appendix B.

Although the bursting of Result 2 is recurrent, it does not appear to be periodic, in the sense that the exact values of t_i, x_i, and $\tilde{h}_i$ of one burst may be quite different from the t_i, x_i, and $\tilde{h}_i$ of another burst. There may exist periodic orbits, but they are unlikely to be stable, since stable orbits tend to show up in simulations, and none were observed. An interesting issue is whether the equation pair may actually exhibit chaotic dynamics. One subtlety in the proof is that the value of μ^* is chosen based on the size of the burst. Thus, although the result guarantees recovery from the first burst, it does not necessarily guarantee recovery from all subsequent bursts.

Perhaps the most serious limitation of Result 2 from a practical viewpoint is that it assumes that the near-end transmission is constant. A more realistic situation is to suppose that v_k is sinusoidal. Accordingly, let $v_k = \sin(\omega k)$. The system (2.6)–(2.7) then becomes

$$\tilde{h}_{k+1} = (1 - \mu x_k^2)\tilde{h}_k - \mu x_k \sin(\omega(k+1)) \tag{3.3}$$

$$x_{k+1} = \alpha\tilde{h}_k x_k + \alpha \sin(\omega(k+1)). \tag{3.4}$$

Simulations of (3.3)–(3.4) show bursting. With α, h, μ, and $\tilde{h}_0$ as in the previous simulation and $\omega = 0.05$, Fig. 7(a) shows the received signal x_k while Fig. 7(b) shows the pole location. The first burst occurs at about 4500 iterations, and the average time between bursts is longer than in the previous case. Fundamentally, however, the behavior is nearly indistinguishable from the bursting of (3.1)–(3.2). The differences are fairly subtle; the long drift of $\tilde{h}$ scallops instead of increasing linearly, the drift of $\tilde{h}$ is driven by the "balanced rate" lemma (Lemma 2 of Appendix A) instead of by the competing rate lemma, and x becomes a sine-like waveform instead of a long slow decay. Nevertheless, the same basic sequence of events occurs to cause bursting very similar to the bursting of Result 2. This can be proven with only slightly more effort.

Result 3 (Bursting with sinusoidal disturbance): Consider the system (3.3) and (3.4) with $\omega = 2\pi/N$ and $N \gg 4$. Suppose there are constants ϵ and t_0 with $\alpha/2 \gg \epsilon > 0$ such that i), ii), iii) hold as in Result 2. Then there is a μ^* such that for every $\mu \in (0, \mu^*)$, there are $t_1 < t_2 < t_3 < t_4 < \infty$ such that 1), 2), 3), and 4) occur just as in Result 2.

Proof: See Appendix B.

Although the analysis of the bursting in Results 2 and 3 does not extend directly to the situation where v_k is characterized by certain stochastic properties, it does provide a framework from which to draw hypotheses. Suppose, for example, that v_k is an independent process. Then there is no correlation between x_k and v_{k+1}. This, the driving term in (4.3) is zero mean, and drift is unlikely. On the other hand, if v_k is dependent, then x_k and v_{k+1} are correlated. The competing rate idea suggests that this would drive $\tilde{h}$ towards instability. If this drive is substantial, then bursting will result. Simulations with white Gaussian v_k do not exhibit bursting, while situations with colored Gaussian v_k (white noise passed through a single pole filter) do exhibit the bursting effect.

One common modification to adaptive algorithms like LMS is the addition of leakage factor. In the linear case (with no feedback of the error into the output), leakage provides a safety net that guarantees exponential stability of the error system. In the echo-cancellation application, however, leakage cannot always prevent bursting. The parameter update with leakage $\lambda \in [0, 1)$ replaces (2.6) with

$$\tilde{h}_{k+1} = (1 - \lambda - \mu x_k^2)\tilde{h}_k - \mu x_k v_{k+1} + \lambda h. \tag{3.5}$$

For the special case $v_k = 1$ and $w_k = 0$, the system (2.7) and (3.5) has an equilibrium at

$$\tilde{h}^* = \frac{\lambda h - \mu x^*}{\lambda + \mu x^{*2}} \tag{3.6}$$

$$x^* = \frac{\alpha}{1 - \alpha\tilde{h}^*}. \tag{3.7}$$

Since the denominator of (3.6) is positive, there are finite solutions to (3.6)–(3.7) for small α. For properly sized λ (such that $|\alpha\tilde{h}^*| < 1$), bursting is unlikely. For leakage too small, however, the continuity of (3.6)–(3.7) shows that the solution must approach the solution for $\lambda = 0$, which is $x^* = 0$, $\tilde{h}^* = \infty$. For such λ, the equilibrium causes (2.7) to be unstable, and bursting will result. Simulations verify that properly scaled λ inhibit bursting, while λ too small allow bursting to occur.

IV. The Double Adaptive Hybrid

When both ends of a phone line are equipped with an adaptive echo canceler as in Fig. 5, the behavior of the system is described by (2.8)–(2.11). If both inputs w_k and v_k are adequately excited, then r_k and x_k are adequately excited (unless w_k and v_k are related in a very singular way). Arguments similar to those used to show the stability of the adequately excited single adaptive hybrid system show that the parameter estimate errors $\tilde{g}$ and $\tilde{h}$ become small. An added subtlety arises in the symmetry of the problem $-v_k$ must be "small" as a disturbance term in (2.8), yet must be "large" as the principle ingredient of the persistent signal r_k in (2.9). Similarly, w_k must be "small" as a disturbance term in (2.9), must be "large" as the principle ingredient of the persistent signal x_k in (2.8). A precise untangling requires a careful juggling of the signals v_k and w_k. A more comprehensive treatment will doubtless need to include frequency effects. See [14] for a discussion.

The situation which led to bursting in the single adaptive hybrid was when the far transmitter was silent. Interestingly, the same conditions do not destabilize the double hybrid. With w_k set to zero to model the quiescent end, (2.8)–(2.11) become

$$\tilde{h}_{k+1} = (1 - \mu x_k^2)\tilde{h}_k - \mu x_k v_{k+1} \tag{4.1}$$

$$\tilde{g}_{k+1} = (1 - \mu r_k^2)\tilde{g}_k \tag{4.2}$$

where

$$x_{k+1} = \tilde{g}_k r_k \tag{4.3}$$

$$r_{k+1} = \tilde{h}_k x_k + v_{k+1}. \tag{4.4}$$

The key to the analysis of (4.1)–(4.4) is to note that if v_k is adequately excited, then r_k is adequately excited, and $\tilde{g} \to 0$ exponentially. Thus the echo at the far end is canceled. This implies that $x_k \to 0$ exponentially and hence that the update of $\tilde{h}$ in (4.1) ceases. In this ideal case, the double adaptive hybrid will not burst.

Result 4 (Stability of underexcited double adaptive hybrid with exact matching): Consider the system (4.1)–(4.4) and suppose that v_k is adequately exciting. Then there are δ_h, δ_g, and μ^* such that for all $|\tilde{h}_0| < \delta_h$, $|\tilde{g}_0| < \delta_g$, and $\mu \in (0, \mu^*)$ imply that $\tilde{g}_k \to 0$ exponentially and $\tilde{h}_k \to h^*$ as $k \to \infty$.

Proof: See [13].

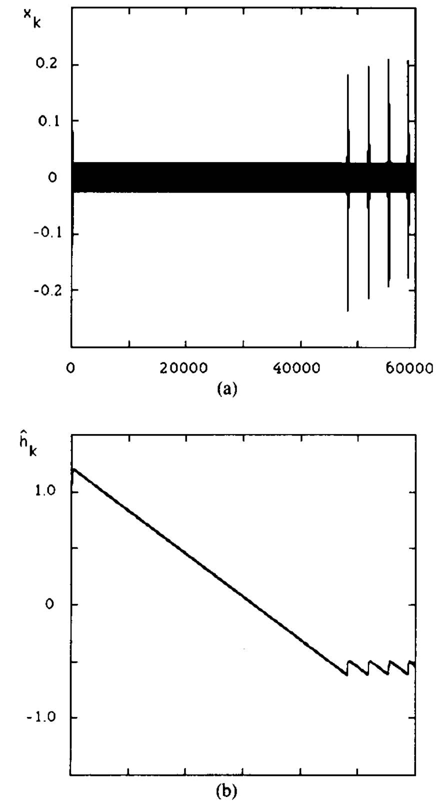

Fig. 8. Bursting in double ended adaptive hybrid with mismodeling. (a) Received signal. (b) Parameter estimate.

The system (4.1)–(4.4) has an equilibrium at $\tilde{g} = 0$ and any value of $\tilde{h}$. The exact value h^* attained depends on the particular sequence v_k and the starting errors $\tilde{h}_0$ and $\tilde{g}_0$. The apparent implication of this result is that adaptive echo cancelers do not burst when used at both ends of the phone line. In the setup of (4.1)–(4.4), the complexity of the adaptive filter is sufficient to exactly match the dynamics of the assumed echo channel. This idealization is unlikely in practice, since real communication channels are subject to distributed effects, small nonlinearities, environmental disturbances, etc.

Suppose that the echo path can be modeled as a second-order moving average with parameters g_1 and g_2 while the adaptive filter has only a single adjustable parameter $\hat{g}_k$. If the input at the near end is a sinusoid $v_k = \sin(\omega k)$, then (4.1)–(4.4) become

$$\tilde{h}_{k+1} = (1 - \mu x_k^2)\tilde{h}_k - \mu x_k \sin(\omega(k+1)) \tag{4.5}$$

$$\tilde{g}_{k+1} = (1 - \mu r_k^2)\tilde{g}_k - \mu g_2 r_k r_{k-1} \tag{4.6}$$

$$x_{k+1} = \tilde{g}_k r_k + g_2 r_{k-1} \tag{4.7}$$

$$r_{k+1} = \tilde{h}_k x_k + \sin(\omega(k+1)) \tag{4.8}$$

where $\tilde{g}_k = g_1 - \hat{g}_k$.

Unlike the previous analysis, x_k does not decay exponentially to zero. In fact, since v_k is a sinusoid, r_k (and hence x_k) are "almost" sinusoidal due to the slow time variation of $\tilde{g}$ and $\tilde{h}$ relative to the frequency of the sinusoid. Thus $r_k r_{k-1}$ is "almost" $\sin^2$ as in the proof of Result 3, and $\tilde{g}$ converges to a small region about $-g_2$. The parameter estimate error $\tilde{h}$ is then simply an integration of $(r_{k-1} - r_{k-2})r_{k+1}$, which imparts a slow drift to $\tilde{h}$, ultimately destabilizing (4.8) in the characteristic bursting manner. Simulations are presented in Fig. 8, with $h = 0.7$, $g_1 = 0.7$, $g_2 = 0.49$, $\omega = 0.05$, $\mu = 2^{-5}$, and $\hat{g}_0 = \hat{h}_0 = 0$. The analysis assumes that $\tilde{h}_{t_0}$ and $\tilde{g}_{t_0}$ are sufficiently small, and then shows that the first burst will invariably occur. Combining (4.7) and (4.8) shows that the "closed-loop system" (from v_k to r_k) is

$$r_{k+1} = \tilde{h}_k \tilde{g}_{k-1} r_{k-1} + \tilde{h}_k g_2 r_{k-2} + v_{k+1}. \tag{4.9}$$

Result 5 (Bursting of underexcited double adaptive hybrid with mismodeling): Consider the system (4.4)–(4.8) with $\omega = 2\pi/N$ and $N \gg 4$. Pick $\epsilon > 0$ small enough so that $-\epsilon < \tilde{h}_0 < 0$ and $\tilde{g}_0 < \epsilon$ imply the stability of (4.9). Then there is a μ^* and a k such that for every $\mu \in (0, \mu^*)$, the closed loop system (4.9) becomes unstable.

Proof: See [13].

V. Conclusion

Bursting was observed in laboratory tests of adaptive hybrids. This bursting was reproduced in simulations in a simplified setting which was then analyzed. The kernel of the bursting phenomenon in echo cancellation lies in a driving term which causes the parameter estimates of the adaptive hybrid to drift linearly, until they eventually destabilize the closed-loop system, causing the burst. The contractive power of the parameter estimator then dominates and the system is restabilized. Following restabilization, drift begins again and the bursting cycle repeats. The structural source of the problem lies in the use of an adaptive filter in a feedback setting.

What can be done to protect echo cancelers from such bursting? Several points deserve comment.

1) The single adaptive hybrid appears to be poorly conditioned in certain operating situations. If no far-end signal is present, the near-end signal is narrow-band, the echo path delay is short, and a doubletalk detector is not present, parameter drift will almost certainly occur. If such an operating situation persists long enough, bursting can occur (Results 2 and 3). Adequate excitation by the input signal at the far end of the line can avert this situation (Result 1).

2) The underexcited double adaptive system with exact matching of the dynamics of the echo path and the adaptive filter cannot burst (Result 4), while even a sight mismatch allows bursting (Result 5).

3) The driving term in the bursting of the double adaptive system has two components: a magnitude term that reflects the quantity of the mismatch, and a phase term that grows larger at higher frequencies. In a practical sense, then, bursting will be less likely to occur in any finite time window if the adaptive filter can closely match the dynamics of the echo path, and if the inputs are bandlimited.

4) Since the heart of the bursting lies in the drift phase, algorithm modifications which are intended to combat drift may decrease the likelihood of bursting. Leakage was briefly examined in a simple example and in simulations. Properly sized leakage appears to protect against bursting, while leakage which is too small did not prevent bursting.

5) What does it mean to persistently (or adequately) excite a pair of adaptive filters when the "input" to one is the "disturbance" to another? The same signal must be "large" as a component of the excitation and "small" as a component of the disturbance. Unlike the single adaptive filter case, there is no obvious "homogeneous" system. This raises a serious question about our current understanding of persistence of excitation of even the simplest adaptive algorithm when used in a nonstandard (feedback) setting.

Appendix A

These lemmas are used in Appendix B to prove the results of the previous sections.

Lemma 1: (The competing rate lemma): Consider the recursion

$$a_{k+1} = (1 - \mu b_k^2)a_k - \mu b_k \quad \text{(A.1)}$$

with $b_k \in [-B, B]$ and $0 < \mu < 1/B^2$.

a) Suppose there is an $\epsilon > 0$ such that $|a_k b_k| > 1 + \epsilon$. Then

$$|a_{k+1}| \leq |a_k| - \mu \in |b_k|.$$

b) Suppose there is an $\epsilon > 0$ such that $|a_k b_k| < 1 - \epsilon$ and sgn $(a_k) = -$sgn (b_k). Then

$$|a_{k+1}| \geq |a_k| + \mu \in |b_k|.$$

Proof: Part a) is proven by writing out the four cases where $a_k > 0$ while $b_k > 0$, $a_k > 0$ while $b_k < 0$, $a_k < 0$ while $b_k > 0$, and $a_k < 0$ while $b_k < 0$. In all four cases, the magnitude of a_k decreases, and statement a) follows. For part b) there are only two cases. □

Remarks: Equation (A.1) should be pictured as having two terms. Part a) provides conditions under which the contraction term $1 - \mu b_k^2$ dominates the driving term $-\mu b_k$, while part b) provides conditions under which the driving term dominates the contraction terms. Parts a) and b) can be easily iterated.

a′) Suppose there is an $\epsilon > 0$ such that $|a_i b_i| > 1 + \epsilon$ for every $i \in [k, k + t]$. Then

$$|a_{k+t+1}| \leq |a_k| - \mu \in \sum_{i=k}^{i=k+t} |b_i|.$$

b′) Suppose there is an $\epsilon > 0$ such that $|a_i b_i| < 1 - \epsilon$ for every $i \in [k, k + t]$. Then

$$|a_{k+t+1}| \geq |a_k| + \mu \in \sum_{i=k}^{i=k+t} |b_i|.$$

The next results require several definitions:

Definition 1: Let a_k be the state of a system with equilibrium $a^* = 0$, and suppose that $\gamma \in (0, 1)$ is such that $|a_{k+1} - a^*| \leq \gamma\,|a_k - a^*|$ for every k. Then the system is said to be *exponentially asymptotically stable* (EAS) and a_k is said to *converge exponentially* to the equilibrium a^*.

Definition 2: Let c_k be a bounded scalar sequence, and suppose there are positive t, c^*, and $c*$ such that $|c_k| > c^*$ for every k, and

$$|c_k| > c * \text{ for some } k \in [j, j+t] \text{ for every } j. \quad \text{(A.2)}$$

Then c_k is said to be *adequately excited.* Note that this is a special case (the one-dimensional version) of the more general definition of persistence of excitation in [3].

Remark: We have decided to use the term "adequate" instead of "persistent" excitation because we believe the use of the latter term is likely to cause considerable confusion in the echo cancellation application. It is well known that the parameter estimates of the LMS adaptive filter converge to a small ball about the desired value when the input is persistently exciting. In the system (2.6)–(2.7), however, x_k (the input) is persistently exiting for LMS, yet the parameter estimates simply drift until $|\alpha \bar{h}|$ becomes larger than 1, no matter what the actual value of the desired parameter h. The reason for this behavior is the feedback path that returns the error as the input. The echo canceler is, therefore, a problem setting in which persistently exciting inputs to the standard LMS algorithm do not guarantee any form of good behavior. A more satisfying definition of persistence of excitation for this problem would involve both external inputs v_k and w_k, as suggested by Example 1.

Lemma 2: (The balanced rate lemma): Consider the recursion

$$a_{k+1} = (1 - \mu\alpha b_k^2)a_k - \mu\beta b_k^2 \quad \text{(A.3)}$$

with α and β positive integers. Suppose also that b_k is bounded by B and is adequately excited and that μ is a positive stepsize less than $2/\alpha B_2$. Then the sequence a_k converges exponentially to the constant $-\beta/A$.

Proof: See [13]. □

Remark: Note that the driving term in (A.3) contains b_k^2 instead of b_k as in (A.1). The name of the lemma arises since the "rate" of the b_k^2 in the contraction term balances the "rate" of the b_k^2 in the driving term, causing (A.3) to converge.

If the input to a linear time invariant (strictly) minimum phase system is persistently excited, then the output of that system is also persistently excited. See [5] for a precise statement. The next lemma shows that the output is adequately excited, even if the plant is allowed to vary, provided that the variation is slow enough and that the input is adequately excited.

Lemma 3: (The slow variation lemma): Consider the following time varying system:

$$a_{k+1} = b_k a_k + c_k \quad \text{(A.4)}$$

and suppose that the related frozen systems

$$a_{k+1} = b_p a_k + c_k \quad \text{(A.5)}$$

are EAS for every p uniformly in p. If c_k is adequately excited as in (A.2) and if $p \overset{\sup}{\geq} p_0 |b_{p+1} - nb_p|$ is small enough for some finite p_0, then a_k is adequately exciting.

Proof: Obtained by combining the proof of Lemma 5.2 of chapter 5 of [5] with the slow variation assumption. □

Appendix B

Proof of Result 1: The slow variation excitation lemma applied to (2.7) [which holds for small μ], shows that if w_k is adequately excited, v^* is small enough, and $\bar{h}_0$ is small, then x_k will be adequately excited. This implies that the homogeneous part of (2.6) is exponentially asymptotically stable, and hence that for small enough v^*, $\bar{h}$ remains small. As v^* approaches zero, the perturbation to the homogeneous system approaches zero, and hence $\bar{h}$ approaches zero. □

Proof of Example 1: The system (2.6) and (2.7) can be linearized about its equilibrium at $-\alpha v/w$, w is

$$A = \begin{Bmatrix} 1 - \mu w^2 \mu v \left(1 - \frac{1}{w}\right) + 2\mu w \\ \alpha w - \frac{\alpha v}{w} \end{Bmatrix}$$

which has eigenvalues at

$$\frac{1}{2}\left\{-\mu w - 1\beta \pm \sqrt{1 + 2\beta + \beta^2 - 2\mu w - 2\mu\alpha v - 4\alpha w v \left(1 - \frac{1}{w}\right) - 8\alpha\mu w^2}\right\}$$

where $\beta = \alpha v/w < 1$. For μ small enough so that $\mu w \ll 1$, the largest eigenvalue can be bounded above by $1 - 1/2\mu w$, since α, β, w, and v are all positive. Thus, A is stable, and the result follows. □

Proof of Result 2: The proof is divided into four sections corresponding to 1)–4) of the statement of the result.

1) The hypothesis i), ii), and iii) imply that $|x_{t_0}\tilde{h}_{t_0}| < 1 - \epsilon$. Due to the small stepsize assumption, the variation in $\tilde{h}$ is slow compared to the time variation in x_k. Thus,

$$x_{t_0+k} = \frac{\alpha}{1+|\alpha\tilde{h}_{t_0+k}|} + e_k \tag{B.1}$$

which e_k is a sum of two parts, a decaying exponential due to initial conditions, and an $O(\mu)$ part from the variation in $\tilde{h}$. This shows that x_{t_0+k} remains within the bounds i) and implies that the driving term of the competing rate lemma applies. From b') of Appendix A,

$$|\tilde{h}_{t_0+k+1}| \geq |\tilde{h}_{t_0}| + \mu \in \sum_{i=t_0}^{t_0+k} |x_i|. \tag{B.2}$$

Thus, there is a finite t_1 such that $|\alpha\tilde{h}_{t_1}| < 1 - \epsilon$.

2) Since x is generated from (3.2), and since the steady-state value of (3.2) for $\alpha\tilde{h} = -1$ is $\alpha/2$, $\alpha\tilde{h}_{t_1} \in [-1-\epsilon, -1+\epsilon]$ implies that x_{t_1} is within $o(\epsilon)$ of $\alpha/2$. Thus, $|x_{t_1}\tilde{h}_{t_1}|$ is within $o(\epsilon)$ of $|\alpha\tilde{h}_{t_1}/2|$ which is approximately 1/2. Hence, the driving term of the competing rate lemma continues to dominate the behavior of $\tilde{h}$, implying that $\tilde{h}$ continues to grow. Thus, there is a k such that $|\alpha\tilde{h}_{t_1+k}| > 1 + \epsilon$. For all such k, (3.2) is unstable, and the magnitude of x increases exponentially. Thus, there is a finite t_2 such that

$$|\alpha\tilde{h}_{t_2}| > 1+\epsilon \tag{B.3}$$

while

$$x_{t_2} > \alpha. \tag{B.4}$$

3) Note that $|x_{t_{2k}}|$ remains larger than α as long as (B.3) holds. Hence, part a') of the competing rate lemma applies, which shows that

$$|\tilde{h}_{t_2+k+1}| \leq |\tilde{h}_{t_2}| - \mu \in \sum_{i=t_2}^{t_2+k} |x_i|, \tag{B.5}$$

which implies that there is a t_3 such that $|x_{t_3}| \gg \alpha$ with $|\alpha\tilde{h}_{t_3}| < 1 - \epsilon$. For the contraction term of the competing rate lemma to apply, it is necessary that $|1 - \mu x_k^2| < 1$ for every k. How large can x grow? Let $|\alpha\tilde{h}_{t_2}| = \beta$. Then $|x_{t_2+k}|$ can be bounded by $(\alpha + \delta)\beta^k$ for some $\delta > 0$, for every $k \in (t_2, t_3)$. Thus, a (liberal) overbound on the maximum value of x_k is $(\alpha + \delta)\beta^{(t_3-t_2)}$, and μ can be chosen appropriately small.

4) After t_3, (3.2) is decaying exponentially, which implies that there is a t_4 such that $\epsilon < |x_{t_4}| < \alpha - \epsilon$ while $|\alpha\tilde{h}_{t_4}| < 1 - \epsilon$. This brings the situation back to step 1). □

Proof of Result 3: 1) If $\tilde{h}$ were fixed, with $|\alpha\tilde{h}| < 1$, (3.4) would be a stable linear system, and x_k would converge exponentially to a sinusoid with frequency ω, gain β, and phase shift ϕ. Hypotheses i), ii), and iii) combined with the small stepsize assumption show that (3.4) is actually a very slowly varying exponentially stable system. Thus, x_k is within $O(\mu)$ of a sinusoid of frequency ω with a phase shift of ϕ.

$$x_k = \beta \sin(\omega k + \phi) + e_k \tag{B.6}$$

where e_k is again the sum of an $O(\mu)$ term and a decaying exponential as in (B.1). Note that β and ϕ are dependent on ω and $\alpha\tilde{h}$, and that $\beta < \alpha$ for $\tilde{h} < 0$. Equation (3.3) can be rewritten

$$\tilde{h}_{k+1} = (1 - \mu\beta^2 \sin^2(\omega k + \phi))\tilde{h}_k - \mu\beta \sin(\omega k + \phi)\sin(\omega(k+1)). \tag{B.7}$$

Let $y_k^2 = \sin^2(\omega k + \phi)$ and

$$d_k = \sin^2(\omega k + \phi) - \sin(\omega k + \phi)\sin(\omega(k+1)+\phi). \tag{B.8}$$

Then (B.7) is

$$\tilde{h}_{k+1} = (1 - \mu\beta^2 y_k^2)\tilde{h}_k - \mu\beta y_k^2 + \mu\beta d_k. \tag{B.9}$$

The homogeneous part of (B.9) [without the $\mu\beta d_k$ term] is precisely of the form (A.3). Hence, the balanced rate lemma applies, showing that the homogeneous part of (B.9) converges exponentially to $-1/\beta$. The perturbation term d_k can be bounded

$$d_k \leq \left|1 - \cos\left(\frac{2\pi}{N} - \phi\right)\right| + \left|\sin\left(\frac{2\pi}{N} - \phi\right)\right| \tag{B.10}$$

which is small for large N and small ϕ. Thus, (B.9) is exponentially stable, and $\tilde{h}$ converges to a small ball about $-1/\beta$. But $\beta < \alpha$, and hence $-\alpha/\beta < -1$, which implies that there is a t_1 such that $|\alpha\tilde{h}_{t_1}| < 1 - \epsilon$.

2) Consider $\alpha\tilde{h}_{t_1} \in [-1-\epsilon, -1+\epsilon]$. x_k is generated from (3.4), which can be iterated and approximated by

$$x_{t_1+k} = (\alpha\tilde{h}_{t_1})^k x_{t_1} + \alpha \sum_{j=1}^{k} (\alpha\tilde{h}_{t_1})^{k-j} \sin(\omega k). \tag{B.11}$$

In the timescale of this step, $\sin(\omega k)$ will be roughly constant for small ω, denote this value by s. Since the steady-state value of β for $\alpha\tilde{h} = -1$ is $\alpha/2$, (B.11) can be approximated by

$$x_{t_1+k} = \frac{\alpha s}{2} h^k + \alpha s \sum_{j=1}^{k} h^{k-j} \tag{B.12}$$

where $h = \alpha\tilde{h}_{t_1}$. Since $\Sigma_{j=1}^{k} h^{k-j} = h^k - 1/h - 1$, x_{t_1+k} is approximately $\alpha s/2$ while $h \in [-1-\epsilon, -1+\epsilon]$. Thus, (3.3) is

$$\tilde{h}_{t_1+k+1} = \tilde{h}_{t_1+k}\left(1 - \mu s^2 \frac{\alpha^2}{4}\right) - \mu s^2 \frac{\alpha}{2} \tag{B.13}$$

which is subject to the competing rate phenomenon exactly as in step 2) of Result 3. Hence, there is a t_2 such that $|\alpha\tilde{h}_{t_2}| > 1 + \epsilon$ and $x_{t_2} > \alpha$.

(3) and (4) occur exactly as in steps 3) and 4) of Result 3. □

Acknowledgment

The authors would like to thank J. Kenney of Tellabs for so clearly answering all of our questions regarding the adaptive hybrid experiments at Tellabs. Thanks are also due to G. Williamson of Cornell University for his careful reading (and refinements) of the theorems and their proofs in the appendixes.

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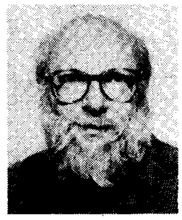

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