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Approximate channel identification via δ-signed correlation

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SUMMARY

A method of approximate channel identification is proposed that is based on a simplification of the correlation estimator. Despite the numerical simplification (no multiplications or additions are required, only comparisons and an accumulator), the performance of the proposed estimator is not significantly worse than that of the standard correlation estimator. A free (user selectable) parameter moves 'smoothly' from a situation with small sum-squared channel estimation error but hard-to-identify channel peaks, to one with a larger sum-squared estimation error but easy-to-identify channel peaks. The proposed estimator is shown to be biased and its behaviour is analysed in a number of situations. Applications of the proposed estimator to sparsity detection, symbol timing recovery and to the initialization of blind equalizers are suggested.

KEY WORDS: signed correlation; channel identification; channel estimation; peak detection; sparsity

1. INTRODUCTION

This paper proposes a method of estimating the channel impulse response by correlating the training sequence with the received data using the sgn_δ function (a signum function with a δ sized 'dead zone'). In many situations, the behaviour of this estimator is analogous to that of a standard correlation channel estimator. Though the estimates are biased, the estimation error is not significantly worse than when using the standard correlation estimator. In addition, the peaks or centres of energy of the channel taps may be more prominently displayed in some of the sgn_δ versions than in the standard correlation estimator.

Many communication channels have a naturally sparse structure (e.g. underwater acoustic communications, wireless communications in a hilly environment) and the channel impulse response has most of its energy concentrated in a few locations [1, 2]. In the digital domain, the sampled channel response has only a few significant tap weights and most of the other tap weights are very small. Performing a channel identification or a channel equalization on such a sparse channel requires filters with long time spans relative to the number of non-zero tap weights. The

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advantages of exploiting sparsity are that less hardware is needed, fewer computations are required in the adaptation of the channel/equalizer coefficients, and there is less misadjustment noise (excess MSE) when an adaptive algorithm is used to track a smaller number of channel/equalizer coefficients [3]. In addition, using fewer taps tends to increase the speed of convergence of the adaptive algorithm [3].

The correlation method of identification [4, 5], FFT-based approaches [6, 7], and least-squares channel estimation [8] (and the recursive implementations such as RLS) are all designed to estimate the complete impulse response of the channel, and cannot be readily simplified to identify just the largest taps of the channel, unless the location of the taps is known *a priori*. Because of this, these methods are computationally complex, requiring from $O(N)$ to $O(N\log(N))$ computations per data symbol. The proposed δ-signed method is closely related to the standard correlation method, but requires no multiplications or additions (other than a simple counter or accumulator). When the channel is sparse (only a few significant taps scattered among a large number of very small taps), or when only the location of the cursor is required, then the problem of identifying which taps are large may be simpler than actually finding all the taps and choosing the largest ones.

The δ-signed correlation method is a low complexity scheme that addresses the problem of sparsity identification and peak detection and may be used in a variety of applications. For example, the rough estimates may help to identify (and hence exploit) sparsity, to initialize an adaptive equalizer, and to track time variations in the sparsity structure. They may also be useful in certain timing and synchronization tasks, and possible applications are detailed in Section 7. This paper proceeds by reviewing the standard correlation estimator in the next section, and introduces the δ-signed correlation estimator in Section 3. The mean of the δ-signed correlation estimator is derived in Section 4. Section 5 analyses the behaviour of the δ-signed correlation estimator in a variety of situations. Simulation results illustrating some of the properties of the proposed estimator are provided in Section 6 and Section 8 concludes.

2. BACKGROUND: THE CORRELATION METHOD

The correlation method is a low complexity channel estimation technique that is suitable when the training sequence is self-orthogonal, i.e. white. The locations of the non-zero portions of the channel response can be determined by correlating the training sequence with the received data for all possible time shifts over a time window that spans the entire duration of the channel impulse response. The peaks of the correlation estimate gives an idea of the time delays at which the channel shows significant energy.

The system model consists of the transmission of a source sequence $u(k)$ through a linear finite-impulse-response (FIR) channel. Let $r(k)$ denote the noisy output of the system represented by the impulse response $h(i)$. Let L be the number of channel impulse response coefficients. Then the received sequence $r(k)$ can be expressed as

$$r(k) = \sum_{i=0}^{L-1} u(k-i)h(i) + w(k) \tag{1}$$

where $w(k)$ is the additive noise sequence at the receiver. If the transmitted source symbols are known (i.e. a training sequence) then the 'correlation estimates' $\hat{h}(i)$ of the channel taps can be

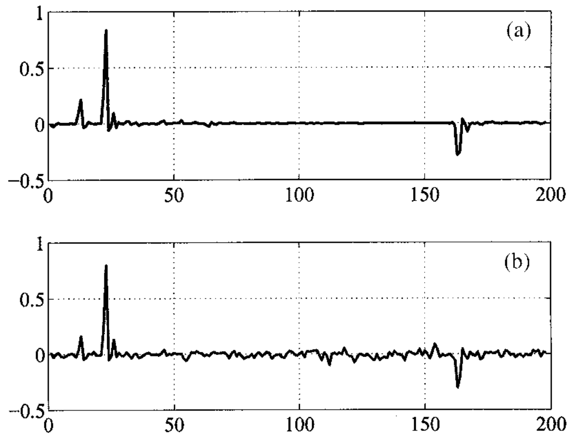

Figure 1. (a) Channel impulse response of a measured sparse channel from the SPIB database. (b) Correlation estimate of the channel impulse response.

calculated as

$$\hat{h}(i) = \frac{1}{N} \sum_{k=1}^{N} u(k) r(k+i) \tag{2}$$

where N is the length of the training sequence. It is well known that under suitable assumptions, namely a self-orthogonal training sequence and a zero-mean uncorrelated noise sequence, this method leads to unbiased estimates of the channel coefficients, with a variance that decays smoothly as the number of points used in the correlation grows (in most cases of interest, this means the length of the training sequence).

For example, Figure 1 shows the channel impulse response of a measured microwave channel taken from the SPIB database [9] and the channel estimates obtained by correlating the training data with the received signal vector. Note that the correlation channel estimator replicates the structure of the channel.

3. THE δ-SIGNED CORRELATION METHOD

As is clear from Equation (2), the correlation method requires N multiplications and N additions for estimating each coefficient of the channel impulse response, leading to an $O(N)$ algorithm. This is computationally prohibitive in many applications, especially in systems operating at high data rates. When using a well-designed training sequence (such as the PN sequence of the 8-VSB HDTV standard), the training sequence consists of $\pm c$ for some constant c, and hence the multiplications can be avoided. However, there is no obvious way to avoid the N additions required for each estimated tap.

The kernel of the idea presented here is to observe that the most important information (in terms of the cross-correlation) is not really contained in the amplitudes of the data, but in its sign.

Hence one can consider using a 'signed correlation'. Formally, consider estimates of the channel impulse response

$$\hat{h}_{\rm sgn}(i) = \frac{1}{N}\sum_{k=1}^{N} {\rm sgn}\{u(k)\}\,{\rm sgn}\{r(k+i)\} \tag{3}$$

where the signum or sign function is defined by

$${\rm sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

In terms of computational complexity, Equation (3) is far superior to Equation (2) since it requires no multiplies, but simply a counter that accumulates how many times the signs agree and disagree. Obviously, $\hat{h}_{\rm sgn}(i)$ differs from $\hat{h}(i)$, but are the differences significant?

This depends on the application. In Reference [10], a variety of autocorrelation methods are considered for use in spectroscopy and radio astronomy, which builds on the earlier work of Reference [11]. Under the assumption that the inputs are Gaussian, the true autocorrelation function can be inferred from that of a signed version, though the number of symbols necessarily increases for a desired level of accuracy. More recently, References [12, 13] have investigated the use of signed correlation in building spectrum analysers for applications in radio astronomical spectroscopy. In communication systems, Willett and Swaszek [14] have studied the performance degradation caused by the use of a signed correlator in the detection problem and van de Beek *et al*. [15] and Hsieh and Wei [16] have investigated the application of quantized correlation for frame synchronization in mobile OFDM systems.

The following intuitive argument suggests how the signed channel estimator might be improved. The most reliable data, from the point of view of the sign function, is that with a large absolute value, since even small amounts of noise may change the sign of data that has a small absolute value. Consequently, it might be possible to improve the estimates by using only those data points that are above some critical threshold δ. Now, consider the estimates

$$\hat{h}_{\delta}(i) = \frac{1}{N}\sum_{k=0}^{N-1} {\rm sgn}\{u(k)\}\,{\rm sgn}_{\delta}\{r(k+i)\} \tag{4}$$

where the function ${\rm sgn}_{\delta}$, is defined by

$${\rm sgn}_{\delta}(x) = \begin{cases} 1 & \text{if } \quad x > \delta \\ 0 & \text{if } -\delta \leqslant x \leqslant \delta \\ -1 & \text{if } \quad x < -\delta \end{cases}$$

Clearly, the signed correlator is a special case of the ${\rm sgn}_{\delta}$ correlator with $\delta = 0$.

4. MEAN OF THE ESTIMATORS

This section derives expressions for the mean of the channel estimates for the signed and δ-signed correlator under the following simplifying assumptions:

- The noise sequence $w(k)$ is zero mean, real, white and Gaussian with variance σ^2.
- The training symbols, $u(k)$ are i.i.d. and are selected from a BPSK constellation.
- The training symbols $u(k)$ are uncorrelated with the noise sequence $w(k)$.

Since the training sequence is BPSK, Equation (3) can be rewritten as

$$\hat{h}_{\mathrm{sgn}}(i) = \frac{1}{N}\sum_{k=1}^{N} \mathrm{sgn}\{u(k)\}\,\mathrm{sgn}\{r(k+i)\}$$

$$= \frac{1}{N}\sum_{k=1}^{N} u(k)\,\mathrm{sgn}\left\{\sum_{m=0}^{L-1} h(m)u(k+i-m) + w(k+i)\right\} \tag{5}$$

Define the sequence $y_i(k)$ as

$$y_i(k) \triangleq \sum_{m=0, m\neq i}^{L-1} h(m)u(k+i-m) \tag{6}$$

and the vectors $\mathbf{b}_{-i}$ and $\mathbf{h}_{-i}$, each of length $L-1$ as

$$\mathbf{b}_{-i} \triangleq [b_0, \ldots, b_{i-1}, b_{i+1}, \ldots, b_{L-1}]^{\mathrm{T}}$$

$$\mathbf{h}_{-i} \triangleq [h(0), \ldots, h(i-1), h(i+1), \ldots, h(L-1)]^{\mathrm{T}} \tag{7}$$

The sequence $y_i(k)$ is a weighted sum of i.i.d. binary random variables and has a discrete probability distribution that takes on a value of 2^{1-L} at each possible $\mathbf{b}_{-i}^{\mathrm{T}}\mathbf{h}_{-i}$, where the vector $\mathbf{b}_{-i} \in \{\pm 1\}^{L-1}$. Since $w(k+i)$ is independent of $y_i(k)$, the probability density function of $\{y_i(k) + w(k+i)\}$ can be computed by convolving the probability densities of $y_i(k)$ and $w(k+i)$. Hence, $\mathrm{sgn}\{r(k+i)\}$ can be expressed as

$$\mathrm{sgn}\left\{\sum_{m=0}^{L-1} h(m)u(k+i-m) + w(k+i)\right\} = \begin{cases} -u(k)\mathrm{sgn}\{h(i)\} & \text{with prob } p(i) \\ u(k)\mathrm{sgn}\{h(i)\} & \text{with prob } 1-p(i) \end{cases} \tag{8}$$

and

$$p(i) = \sum_{\mathbf{b}_{-i}\in\{\pm 1\}^{L-1}} 2^{-L+1}\,\mathcal{Q}\left\{\frac{|h(i)| + \mathbf{b}_{-i}^{\mathrm{T}}\mathbf{h}_{-i}}{\sigma}\right\} \tag{9}$$

where $\mathcal{Q}$ is the error function defined as

$$\mathcal{Q}(x) = \frac{1}{\sqrt{(2\pi)}} \int_x^\infty \exp\left(-\frac{y^2}{2}\right) \mathrm{d}y \tag{10}$$

Hence, the mean of the channel estimate for the signed correlator is given by

$$E[\hat{h}_{\mathrm{sgn}}(i)] = \mathrm{sgn}(h(i)) \left[1 - \sum_{\mathbf{b}_{-i} \in \{\pm 1\}^{L-1}} 2^{-L+2} \mathcal{Q}\left\{ \frac{|h(i)| + \mathbf{b}^{\mathrm{T}}_{-1} \mathbf{h}_{-i}}{\sigma} \right\} \right] \tag{11}$$

The mean of the channel estimates for the δ-signed correlator can be similarly derived and it is given by

$$E[\hat{h}_\delta(i)] = \mathrm{sgn}(h(i)) \left[1 - \sum_{\substack{\mathbf{b}_{-i} \in \{\pm 1\}^{L-1} \\ b_L \in \{\pm 1\}}} 2^{-L+1} \mathcal{Q}\left\{ \frac{|h(i)| + \mathbf{b}^{\mathrm{T}}_{-i} \mathbf{h}_{-i} + b_L \delta}{\sigma} \right\} \right] \tag{12}$$

Note that the means of the signed and the δ-signed correlation channel estimators are upper-bounded by unity, and hence these estimators are biased.

Theorem 1 (*Order preservation*)
The δ-signed correlation channel estimator preserves the order of the channel coefficients in the mean, i.e., if $|h(m_0)| \geqslant |h(m_1)| \geqslant \cdots \geqslant |h(m_{L-1})|$ then $|E[\hat{h}_\delta(m_0)]| \geqslant |E[\hat{h}_\delta(m_1)]| \geqslant \cdots \geqslant |E[\hat{h}_\delta(m_{L-1})]|$.

Proof. Let $|h(m_1)| \geqslant |h(m_2)|$ for some choice of m_1 and m_2. Observe that from Equation (12),

$$|E[\hat{h}_\delta(m_1)]| = \left[1 - \sum_{\substack{\mathbf{b}_{-m_1} \in \langle \pm \mathbf{1} \rangle^{L-1} \\ b_L \in \langle \pm \mathbf{1} \rangle}} 2^{-L+1} \mathcal{Q}\left\{ \frac{|h(m_1)| + \mathbf{b}^{\mathrm{T}}_{-m_1} \mathbf{h}_{-m_1} + b_L \delta}{\sigma} \right\} \right] \tag{13}$$

Comparing the terms on the right-hand side for $|E[\hat{h}_\delta(m_1)]|$ and $|E[\hat{h}_\delta(m_2)]|$, notice that for any choice of values of $b_0 \in \{\pm 1\}, \ldots, b_L \in \{\pm 1\}$,

$$|h(m_1)| \pm |h(m_2)| + b_\delta \delta + \sum_{\substack{k=0, k \neq m_1 \\ k \neq m_2}}^{K=L-1} b_k h(k)$$

$$\geqslant |h(m_2)| \pm |h(m_1)| + b_\delta \delta + \sum_{\substack{k=0, k \neq m_1 \\ k \neq m_2}}^{K=L-1} b_k h(k) \tag{14}$$

$\mathcal{Q}(x)$ is a monotonically decreasing function of x and therefore,

$$-\mathcal{Q}\left\{\frac{1}{\sigma}\left(|h(m_1)| \pm |h(m_2)| + b_\delta \delta + \sum_{\substack{k=0, k \neq m_1 \\ k \neq m_2}}^{K=L-1} b_k h(k)\right)\right\}$$

$$\geqslant -\mathcal{Q}\left\{\frac{1}{\sigma}\left(|h(m_2)| \pm |h(m_1)| + b_\delta \delta + \sum_{\substack{k=0, k \neq m_1 \\ k \neq m_2}}^{K=L-1} b_k h(k)\right)\right\} \tag{15}$$

Accordingly,

$$E[\hat{h}_\delta(m_1)] \geqslant E[\hat{h}_\delta(m_2)] \tag{16}$$

Thus, on average, the tap estimates from the δ-signed correlation are ranked in the same order as the taps of the channel impulse response. Since the signed-correlation channel estimator is a special case of the δ-signed correlator, it also satisfies the order preservation property.

5. ASYMPTOTIC ANALYSIS

In an effort to understand the behaviour of the signed and δ-signed correlation estimators, this section considers the special cases when $\sigma \to 0$ and $\sigma \to \infty$. Since the signed and the δ-signed correlation estimators are typically biased, we focus attention on the relative magnitudes of the channel tap coefficients.

5.1. 2-tap channel

Consider a channel impulse response with only two taps. Without loss of generality, assume that $|h(0)| \geqslant |h(1)|$. Define the ratio of the magnitudes of the tap coefficients as Γ, where

$$\Gamma = \frac{|h(0)|}{|h(1)|} \geqslant 1 \tag{17}$$

Further define

$$\hat{\Gamma}_\delta \triangleq \frac{|E[\hat{h}_\delta(0)]|}{|E[\hat{h}_\delta(1)]|} \tag{18}$$

Recall from Equation (12) that

$$[E[\hat{h}_\delta(0)]| = 1 - \sum_{b_1, b_2 \in \{\pm 1\}} 2^{-1} \mathcal{Q}\left\{\frac{|h(0)| + b_1 h(1) + b_2 \delta}{\sigma}\right\} \tag{19}$$

Using the approximation

$$\mathcal{Q}(x) \approx \frac{1}{2} - \frac{x}{\sqrt{2\pi}}, \quad \text{if } x \ll 1 \tag{20}$$

it can be shown that

$$\lim_{\sigma \to \infty} |E[\hat{h}_\delta(0)]| = \frac{2|h(0)|}{\sqrt{2\pi\sigma}} \tag{21}$$

and hence

$$\lim_{\sigma \to \infty} \hat{\Gamma}_\delta = \Gamma \tag{22}$$

In the noiseless scenario (when $\sigma \to 0$) recall that

$$\lim_{\sigma \to 0} \mathcal{Q}\left(\frac{x}{\sigma}\right) = \begin{cases} 1 & \text{if } x < 0 \\ 0{\cdot}5 & \text{if } x = 0 \\ 0 & \text{if } x > 0 \end{cases} \tag{23}$$

and hence

$$\lim_{\sigma \to 0} \hat{\Gamma}_\delta = \begin{cases} \infty & \text{if } \delta < |h(0)| - |h(1)| \\ 3 & \text{if } \delta = |h(0)| - |h(1)| \\ 1 & \text{if } \delta > |h(0)| - |h(1)| \end{cases} \tag{24}$$

For example, consider a 2-tap channel with an impulse response of $\mathbf{h} = [0{\cdot}8,\ 0{\cdot}6]$. Figure 5 plots the ratio of the magnitudes of the mean of the δ-signed correlation estimator, as a function of the SNR, for various values of the threshold δ. From the plot, it is clear that at high values of SNR, the tap with the larger magnitude is greatly enhanced as long as the value of the threshold $\delta \leqslant |h(0) - |h(1)|$. Such a property of the δ-signed estimator would be useful in an application where only the location of cursor is required.

5.2. *L-tap channel*

Now consider a channel impulse response with L taps. If L is large it is possible to employ the central limit theorem and approximate the probability density function of $\{y_i(k) + w(k+i)\}$ (recall Equation (6)) as a zero-mean, Gaussian distribution. Formally, this requires the satisfaction of the Lindeberg–Feller condition (see the appendix for details). For the present purposes, we assume that the Gaussian approximation holds, and that the channel is unit norm, i.e. $\|\mathbf{h}\|^2 = 1$. Hence,

$$[y_i(k) + w(k+i)] \sim \mathcal{N}(0, \sigma_i^2) \tag{25}$$

where

$$\sigma_i^2 = 1 - |h(i)|^2 + \sigma^2 \tag{26}$$

Under this approximation, Equation (9) simplifies to

$$p(i) = \mathcal{Q}\left\{\frac{|h(i)|}{\sigma_i}\right\} \tag{27}$$

and the mean of the signed correlation estimator simplifies to

$$E[\hat{h}_{\mathrm{sgn}}(i)] = \mathrm{sgn}\{h(i)\}\left[1 - 2\mathcal{Q}\left\{\frac{|h(i)|}{\sqrt{1 - |h(i)|^2 + \sigma^2}}\right\}\right] \tag{28}$$

A similar analysis for the δ-signed correlator results in

$$E[\hat{h}_{\delta}(i)] = \mathrm{sgn}\{h(i)\}\left[1 - \mathcal{Q}\left\{\frac{|h(i)| + \delta}{\sqrt{1 - |h(i)|^2 + \sigma^2}}\right\} - \mathcal{Q}\left\{\frac{|h(i)| - \delta}{\sqrt{1 - |h(i)|^2 + \sigma^2}}\right\}\right] \tag{29}$$

Using the approximation in Equation (20) implies that

$$\lim_{\sigma \to \infty} \frac{|E[\hat{h}_{\delta}(i)]|}{|E[\hat{h}_{\delta}(j)]|} = \frac{|h(i)|}{|h(j)|} \tag{30}$$

Hence, at low values of SNR, the δ-signed correlation estimator preserves the relative magnitudes of the channel taps in the mean.

Theorem 2 (*Dominant tap enhancement*)
If the channel impulse response coefficients are such that $|h(i)| \geqslant |h(j)|$ and $|h(i)| \leqslant \sqrt{2(1+\sigma^2)/3}$, then the signed correlation channel tap estimate of the L-tap channel, under the Gaussian approximation, satisfies

$$\hat{\Gamma}_{ij}^{\mathrm{sgn}} \triangleq \frac{|E[\hat{h}_{\mathrm{sgn}}(i)]|}{|E[\hat{h}_{\mathrm{sgn}}(j)]|} \geqslant \frac{|h(i)|}{|h(j)|} \triangleq \Gamma_{ij} \tag{31}$$

Proof. Define the function

$$f(x) \triangleq 1 - 2\mathcal{Q}\left\{\frac{x}{\sqrt{1 - x^2 + \sigma^2}}\right\}, \quad \forall x \in [0, 1) \tag{32}$$

This is the magnitude of the mean of the signed-correlation estimator of Equation (28). It can be easily shown that the function $f(x)$ satisfies the property

$$f''(x) \geqslant 0 \quad \text{if } 0 \leqslant x \leqslant \sqrt{\frac{2(1+\sigma^2)}{3}} \tag{33}$$

Hence, $f'(x)$ is a monotonically increasing function of x. Consider some $x_1, x_2 \in [0, 1)$ such that

$$0 \leqslant x_1 \leqslant x_2 \leqslant \sqrt{\frac{2(1+\sigma^2)}{3}} \tag{34}$$

Applying the mean value theorem to the function $f(x)$ gives

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(\zeta_2), \quad \zeta_2 \in [x_1, x_2] \tag{35}$$

and

$$\frac{f(x_1) - f(0)}{x_1 - 0} = f'(\zeta_1), \quad \zeta_1 \in [0, x_1] \tag{36}$$

Due to the monotonicity of $f'(x)$, $f'(\zeta_2) \geqslant f'(\zeta_1)$ and since $f(0) = 0$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} \geqslant \frac{f(x_1) - f(0)}{x_1 - 0}$$

$$\Rightarrow \frac{f(x_2)}{f(x_1)} \geqslant \frac{x_2}{x_1} \tag{37}$$

Hence, the dominant tap is augmented compared to the smaller taps, which reinforces the observations from the example in Figures 3 and 4 that the taps with the largest energy are more clearly visible in the simpler δ-signed correlator than in the standard correlator. Note that this property of the δ-signed correlation estimator makes it suitable for applications like sparsity detection, where the aim is to determine the locations of the significant taps of the channel impulse response.

6. SIMULATION RESULTS

The performance of the signed and δ-signed correlation estimators were studied using simulations. The channel impulse response (see Figure 1) of a measured microwave channel [9] was

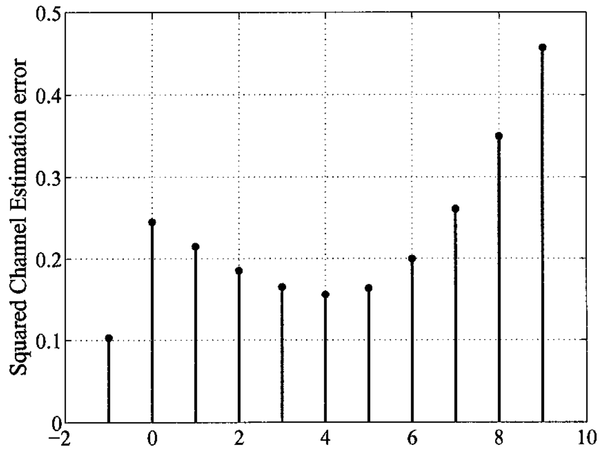

Figure 2. Averaged sum-squared channel estimation error as a function of δ. The abscissa '-1' corresponds to the standard correlation estimator. The abscissas '0' to '9' correspond to δ-signed estimators as δ takes on values between 0 to 1.8λ in steps of 0.2λ each.

chosen for the simulations. The data set was drawn from a BPSK source constellation. A 511-length PN sequence, as defined by the high definition television (HDTV) standards [17], was chosen as the training data. The SNR at the receiver was assumed to be 15 dB. Since the δ-signed channel estimator is biased, the estimates were normalized such that the estimated channel impulse response was of unit norm, i.e. $\|\hat{\mathbf{h}}_\delta\|^2 = 1$.

Figure 2 compares the averaged sum-squared channel estimation error of the δ-signed estimator, for various choices of δ, with the performance of the standard correlation estimator. The abscissa of '-1' in the figure corresponds to the correlation estimator, while abscissas between '0' and '9' represents the estimation error as δ varies from 0 to 1.8λ in steps of 0.2λ each. The value of λ is chosen to be the mean of the absolute value of the received signal vector, i.e. $\lambda = E[|r(n)|]$. Although the performance of the δ-signed correlator is worse than the standard correlation estimator, the degradation is small and is only in the order of a couple of dB. Furthermore, the performance of the δ-signed estimator improves with increasing values of δ up to a point and this suggests that there exists some optimal value for the choice of δ. For this particular example, 0.8λ seems to be the optimal choice of δ. However, the value of the optimal δ may be a function of the channel and hence difficult to predict.

Figures 3 and 4 show estimates of the same channel for a number of different values of δ. Figure 3 shows the channel estimates for δ values of 0 in (a) and 0.4λ in (b). Figure 4 continues with 0.8λ in (a) and 1.4λ in (b). The signed version ($\delta = 0$) clearly shows the location of the cursor but much of the channel detail is lost, due to the coarseness of the sign function. As δ grows, the three peaks of the channel become more and more prominent. Especially for the choice of $\delta = 1.4\lambda$, although the channel estimates are very noisy, the three peaks of the channel are quite prominent (see Figure 4) and hence suitable for sparsity detection. For yet larger values of δ (not shown), there are not enough non-zero points to register and the estimates degrade. This example suggests that careful selection of δ may allow the simpler estimator to retain certain details of the channel despite the numerical simplifications.

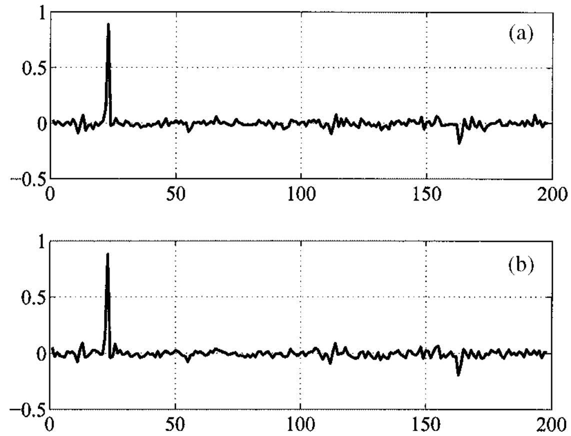

Figure 3. The estimated channel impulse response using the signed correlation method in (a), and using the δ-signed correlation method for a value of $\delta = 0.4\lambda$ in (b).

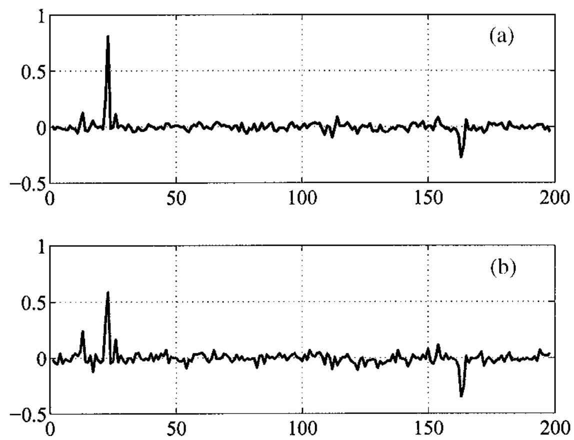

Figure 4. The estimated channel impulse response using δ-signed correlation method for δ values of 0.8λ in (a) and 1.4λ in (b).

7. USING THESE ROUGH ESTIMATES

A nice feature of the δ-signed correlation estimator is that the peaks of the channel are more prominent for larger δ. This is especially useful when the energy in the taps is sparse, since the rough channel estimates $\hat{h}_\delta(i)$ can be used to identify where the channel energy is. There are several possible benefits to having a simple approximate impulse response available.

7.1. Use in initialization

The approximate channel estimates given by the δ-signed correlator can be used to initialize a blind adaptive equalizer (or a decision feedback equalizer, if the estimate is accurate enough). The taps that follow the cursor (generally, the largest peak) can be used directly to initialize (for

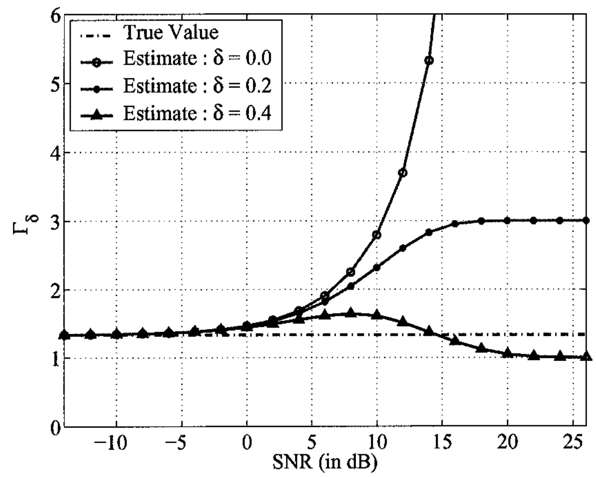

Figure 5. The ratio of the magnitudes of the mean of the δ-signed correlation estimator for a 2-tap channel, $\mathbf{h} = [0.8, 0.6]$ as a function of the SNR for various values of δ. ·-·-·-, True value; —○—, estimate: $\delta = 0.0$; —●—, estimate: $\delta = 0.2$; —▲—, estimate: $\delta = 0.4$.

instance) a blind infinite-impulse-response (IIR) equalizer. As shown in Reference [18] if this initialization is good enough to reduce the SINR to about -3.8 dB, it can be guaranteed that the constant modulus algorithm (CMA) will converge to a minimum corresponding to a delay consistent with the initialization. In particular, this guarantees that no undesirable saddle points will be encountered. Of course, even if the rough estimate is not good enough to manage to reduce the SINR this far, it is intuitively reasonable that the initialization will be far better than an all zero 'cold start' of the equalizer parameters.

7.2. Use in detecting and exploiting sparsity

Many common communication channels have a sparse structure [1], that is, their impulse responses contain bursts of activity, followed by brief quiescent moments, followed by further bursts of activity. In designing equalizers (whether linear, DFE, or others), it may be advantageous to exploit this sparsity.

The rough channel estimates provided by the δ-signed correlator can be used to detect the presence of sparsity in the channel. By setting an appropriate threshold, all taps below this threshold may be assumed insignificant (set to zero). Consider, for instance, a threshold somewhere above the noise floor in Figure 4(a). The remaining non-zero taps are located around the three peaks. Exploiting this sparsity is as simple as initializing and adapting only those parameters in or around these non-zero regions.

7.3. Use in time-varying situations

When sparse adaptation is being done in the equalizer, typically only a subset of the taps are actually adapted. This has several benefits including a reduced noise floor, and faster adaptation (than when adapting all the taps). One problem, however, is that in a time-varying situations the locations of the significant channel taps may change. Usually, it is not possible to detect this just by looking at the motion of the adapted equalizer parameters. A low complexity solution to this problem is to use the δ-signed channel estimator. One can adapt the sparse equalizer as before—but when the rough channel estimates indicate energy at a location where there are no taps in the equalizer, then these taps can be adapted.

7.4. Ability to exploit field sync

It is not necessary to correlate the received signal only with the official 'training sequence'. In the 8-VSB HDTV standard [17], for instance, there are also 'field sync' signals which are interspersed throughout the data record. Observe that the signal $u(k)$ in Equations (2)–(4) need not represent consecutive symbols. They need only be matched up (with correct delays) to the received signal. This may be useful since all the estimators give better results for larger N. Of course, it is also possible to have $u(k)$ representing several consecutive sets of training data in order to achieve a better estimate. This extension is limited by the desire to use the rough estimates to track or follow channel time variations.

7.5. Help in timing and synchronization

Finally, some kind of correlation is often done to look for the start of each frame. Monitoring the time difference between successive peaks in the correlation (i.e. between successive frames) and dividing by the number of symbols expected in that frame, gives a measure of clock timing. For example, Kim *et al.* [19] describes a timing recovery technique based on detecting the 'field sync' signals in the HDTV data record and using some sort of phase locked loop (PLL). As illustrated in Section 6 the δ-signed correlation estimator is especially well suited for detecting the peaks and hence can be used for timing recovery.

8. SUMMARY

This paper has proposed a method of correlating the training sequence with the received data using the sgn_δ function (a signum function with a δ sized 'dead zone'). The δ-signed correlation estimator was shown to be biased and its behaviour was analysed in a number of situations. The performance of the δ-signed correlation estimator was found to be only marginally worse than that of the correlation estimator. It was further shown that the δ-signed correlation estimator displays the peaks in the channel impulse response more prominently than the correlation estimator, for suitable choices of δ. This property of the δ-signed correlation estimator makes it suitable for applications like peak detection and sparsity detection. In scenarios where the use of a correlation procedure would be useful, but where computational complexity is of concern, it may be worthwhile to consider using a δ-signed correlation.

APPENDIX

Lindeberg–Feller condition: Let the random variables $X_n, n \geqslant 1$ be independent (but not necessarily identically distributed) and suppose $E(X_k) = 0$ and $\text{Var}(X_k) = \sigma_k^2$. Define

$$s_n^2 \triangleq \sigma_1^2 + \cdots + \sigma_n^2 = \text{Var}\left(\sum_{i=1}^{n} X_i\right)$$

Then X_k satisfies the Lindeberg condition [20] if for all $t > 0$

$$\frac{1}{s_n^2}\sum_{k=1}^{n} E[X_k^2 1_{[|X_k/s_n|>t]}] \to 0 \tag{A1}$$

as $n \to \infty$, which implies that

$$\frac{S_n}{s_n} \Rightarrow \mathcal{N}(0, 1) \tag{A2}$$

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